Challenge Problems Appendix

1. Write an equation that forms a system of equations with $x + y = 4$, so that the system has

a) no solution

b) infinitely many solutions

c) one solution

2. Sketch a graph to represent a system of two equations with one solution, so that the two lines have

a) different x-intercepts and different y-intercepts

b) the same x-intercepts but different y-intercepts

c) different x-intercepts but the same y-intercept

d) the same x-intercept and the same y-intercept

3. Find the point of intersection of each linear system.

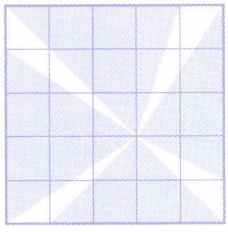
a) $2(x - 4) + y = 6$
$3x - 2(y - 3) = 13$

b) $2(x - 1) - 3(y - 3) = 0$
$3(x + 2) - (y - 7) = 20$

c) $2(3x - 1) - (y + 4) + 7 = 0$
$4(1 - 2x) - 3(3 - y) + 12 = 0$

d) $2(x - 1) - 4(2y + 1) = -1$
$x + 3(3y + 2) = 2$

4. In the figure, the side of each small square represents 1 cm. Determine the area of the shaded part.



5. Solve the following linear system for x and y by letting $a = \frac{1}{x}$ and $b = \frac{1}{y}$.

$$\frac{1}{x} + \frac{3}{y} = \frac{3}{4}$$
$$\frac{3}{x} - \frac{2}{y} = \frac{5}{12}$$

6. Write a linear system in the form

$Ax + By = C$
$Dx + Ey = F$

where A, B, C, D, E, and F are integers and the point of intersection is $(6, -5)$.

7. Each letter represents a different integer. The sums of the columns and two of the rows are shown. Find the two missing row sums.

A	B	A	B	**36**
A	C	A	C	**34**
B	B	B	C	**?**
D	E	F	A	**?**
37	**40**	**41**	**32**	

8. Greg drove from his home in Point Alexander to Belleville at an average speed of 75 km/h. From her home in Chalk River, Claire drove the 18 km to Point Alexander and continued on to Belleville at an average speed of 85 km/h. Greg and Claire left home at the same time.

a) After what length of time did Claire pass Greg?

b) How far were they from Point Alexander when Claire passed Greg?

9. Determine the next three numbers in each sequence.

a) 6, 12, 11, 18, 17, ...

b) 2, 4, 6, 10, 16, 26, ...

c) 1, 2, 5, 10, 17, 26, ...

10. The endpoints of AB are A$(3, -4)$ and B$(11, b)$. The midpoint of AB is M$(n, 3)$. Find the values of b and n.

11. The base of an isosceles triangle lies on the x-axis. The coordinates of the midpoints of the equal sides of the triangle are $(2, 3)$ and $(-2, 3)$. What are the coordinates of the vertices of the triangle?

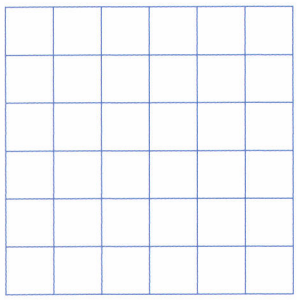
12. A quadrilateral has vertices P$(-3, 1)$, Q$(-1, -5)$, R$(11, 1)$, and S$(1, 3)$. Verify each property.

a) PQRS is a trapezoid.

b) The line segment joining the midpoints of the two non-parallel sides is parallel to the bases.

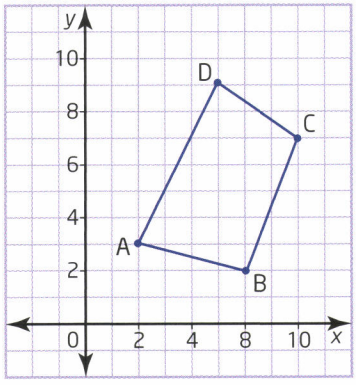
c) The line segment from part b) is half as long as the sum of the lengths of the bases.

13. Copy the grid. Draw six circles in six of the small squares, so that none of the circles is in the same row, column, or diagonal.



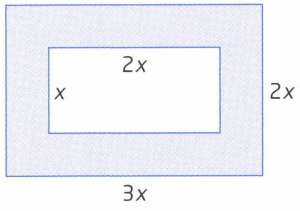
14. Determine the area of the trapezoid with vertices A(−4, 3), B(−1, 4), C(10, 1), and D(−5, −4).

15. Find the area of quadrilateral ABCD.



16. The first time Rohan and Mark raced 20 km on their bicycles, Mark was 2 km from the finish line when Rohan reached it. The next time they raced, Rohan agreed to start 2 km behind Mark. So, Rohan rode 22 km while Mark rode 20 km. If each person went at the same average speed as they did during their first race, who won the second race?

17. The area of the shaded region is 400 cm^2. Find the dimensions of the larger rectangle.



18. Use finite differences to determine whether the data follow a linear relation, a quadratic relation, or neither. For the linear data, find the equation of the line. For the quadratic data, use a graphing calculator to find the quadratic relation.

a)

x	y
−2	−8
−1	−5
0	−2
1	1
2	4
3	7
4	10

b)

x	y
−1	1
0	4
1	1
2	−8
3	−23
4	−44
5	−71

c)

x	y
−5	0
−4	−3
−3	−4
−2	−3
−1	0
0	5
1	12

d)

x	y
0	0
1	1
2	3
3	7
4	15
5	31
6	63

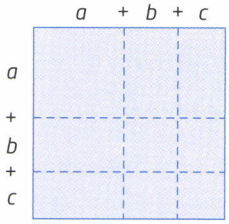
19. The path of a heavy object thrown into the air is modelled by the equation $h = -0.05d^2 + 0.3d + 0.8$, where h represents the height of the object above the ground and d represents the horizontal distance the object travels, both in metres.

a) From what height was the object thrown?

b) How far does the object travel horizontally before it hits the ground?

c) What was the maximum height of the object?

20. The side length of the square is $a + b + c$.



a) Write, expand, and simplify an expression for the area of the square.

b) Use your result from part a) to expand $(2x + 3y + 1)^2$.

21. Factor.

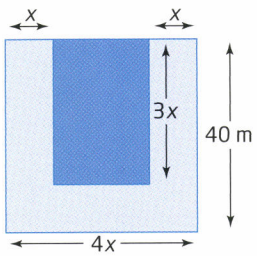
a) $x^4 + 2x^2 + 1$

b) $x^4 + x^2 - 6$

c) $x^4 - 3x^2 - 10$

d) $x^4 + 10x^2y + 9y^2$

22. A small shopping plaza has a U-shaped building with a rectangular parking lot in the middle, as shown. The floor area of the building is 896 m^2.



a) Determine the length of x, in metres.

b) If each parking space requires an area of 24 m^2, how many cars can park in this parking lot?

23. The number 12 is the first *abundant* number. It is the least whole number for which the sum of its factors, not including itself, is greater than itself.

$1 + 2 + 3 + 4 + 6 = 16$

Find the next three abundant numbers.

24. Factor.

a) $2x^4 + 3x^2 + 1$

b) $2x^4 + 5x^2 - 3$

c) $3x^4 - x^2 - 4$

d) $6x^4 - 13x^2 + 6$

e) $2x^4 + 5x^2y + 2y^2$

f) $3x^4 + 11x^2y - 4y^2$

25. The area of a square is represented by the expression $49 - 28a + 4a^2$, where a is a natural number. What are the possible values for the perimeter of the square?

26. Write and simplify the equation obtained when the given expression is substituted for x.

a) $y = x^2 + 1; x = m - 1$

b) $y = 2x^2 - 3; x = 2k + 1$

c) $y = x^2 + 4x - 1; x = 3t - 1$

d) $y = 3x^2 - 2x + 4; x = 3 - 2w$

27. The length, width, and height of a rectangular prism measure whole numbers of centimetres. Three faces of the prism have areas of 144 cm^2, 72 cm^2, and 32 cm^2. What are the dimensions of the prism?

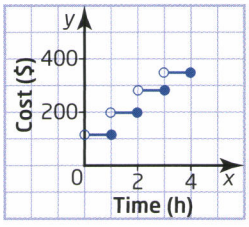
28. Pam and Theo opened a coffee bar. The business lost \$2100 in the first month and \$1900 in the second month. As they hoped, the business continued to improve by \$220/month for the first 2 years.

a) Make a scatter plot showing their financial position at the end of each month.

b) In which month did they start to make a profit?

c) Find an equation to represent the relation between their profit and the number of months since they opened the coffee shop.

29. The cost of renting an ice rink includes an initial fee, plus an additional fee for each hour or part of an hour of use. The graph shows how cost is related to time. Up to and including 1 h costs \$120, greater than 1 h and up to and including 2 h costs \$200 and so on.



a) Explain why open and closed dots are used on the graph.

b) If you were charged \$360, for how many hours did you rent the rink?

c) If you want to rent the rink for 1.5 h, how much will it cost?

30. A photocopier was set to enlarge an original to 150% of its dimensions. A diagram of a rectangle was enlarged and then the image was enlarged again at the same setting. If the final rectangle measured 20.25 cm by 13.5 cm, what were the dimensions of the rectangle on the original diagram?

31. Find a and k so that the quadratic relation $y = ax^2 + k$ passes through each pair of points.

a) $(-3, 11)$ and $(4, 18)$

b) $(-1, -2)$ and $(3, -10)$

c) $(2, 5)$ and $(1, -1)$

d) $(2, 2)$ and $(-4, -4)$

32. Determine an equation for the parabola with the given vertex and passing through the given point.

a) vertex $(-4, -5)$, passing through $(-2, -1)$

b) vertex $(1, 6)$, passing through $(3, 2)$

c) vertex $(-2, 3)$, passing through $(-1, 6)$

33. Find the values of a and k so that the given points lie on the parabola.

a) $y = a(x - 1)^2 + k$; $(2, 6)$, $(3, 12)$

b) $y = a(x + 3)^2 + k$; $(-5, -8)$, $(1, 20)$

c) $y = a(x - 4)^2 + k$; $(1, -13)$, $(-1, -45)$

34. For the relation $y = -2x^2 + 8x + k$, what value(s) of k will result in

a) one x-intercept?

b) two x-intercepts?

c) no x-intercepts?

35. Evaluate each expression. Write answers with positive exponents.

a) $\dfrac{2^{-3} \times 2^{-4}}{2^{-2}}$

b) $\dfrac{5^{-3} \times 5^{-1}}{5^{2}} \times \dfrac{5^{-1} \times 5^{3}}{5^{-2}}$

c) $\dfrac{3^{-3} \times 27^{-1}}{9^{-1} \times 81^{-2}}$

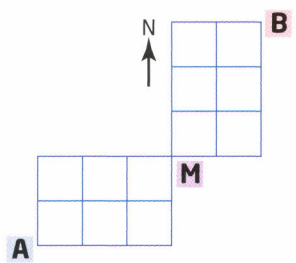
36. Simplify each expression. Write answers with positive exponents.

a) $3x^{-2} \times (3x^{-2})^{-2}$

b) $\dfrac{5x^{-8} \times 3^{-2}x^{3}}{3x^{-4}} \times \dfrac{x^{-4}}{5x^{-2}}$

c) $\dfrac{4x^{2}y^{-3} \times 2^{-2}x^{-4}y^{-2}}{8x^{-2}y^{-2}}$

37. A town is separated into two parts by a river that is crossed by a bridge at M. How many routes are there for a bus to get from A to B if the bus only travels north and east?



38. Consider the equation $x^2 + bx = 0$.

a) For what value of b will the equation have two equal real roots?

b) If the equation has two distinct real roots, what value must one of them have? How is the other root related to b?

39. a) Write a quadratic equation in the form $ax^2 + bx + c = 0$ whose roots are -2 and 3.

b) Is it possible to write another quadratic equation with the same roots? Explain.

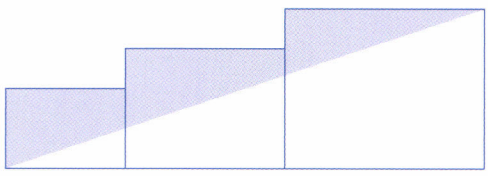
40. If -3 is one root of the equation $3x^2 + bx + 3 = 0$,

a) what is the value of b?

b) what is the other root?

41. The difference between the squares of two consecutive whole numbers is 63. What are the numbers?

42. The smallest rectangle shown is 3 m wide and 2 m tall. The next rectangle is 4 m wide and 3 m tall. The largest rectangle is 5 m wide and 4 m tall. Find the area of the shaded region.



43. A cylinder has a height of 5 cm and a surface area of 100 cm^2. Find the radius of the cylinder, to the nearest tenth of a centimetre.

44. Describe the roots of the equation $ax^2 + bx + c = 0$ in each case. Explain your reasoning and give examples to support your answers.

a) $b^2 - 4ac < 0$

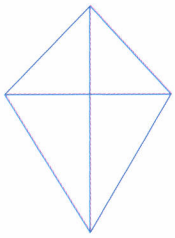
b) $b^2 - 4ac = 0$

c) $b^2 - 4ac$ is a perfect square

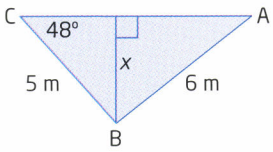
d) $b^2 - 4ac > 0$ but is not a perfect square

45. The areas of two similar triangles are 28 cm^2 and 112 cm^2. What is the ratio of the lengths of their corresponding sides?

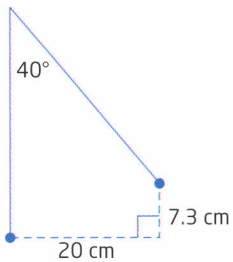
46. The diagonals of a kite measure 10 cm and 12 cm. What is the area of the kite?



47. Find the length of x, to the nearest tenth of a metre, and the measure of $\angle A$, to the nearest degree.



48. When a pendulum swings 40° from the vertical, its lower end moves 20 cm horizontally and 7.3 cm vertically. What is the length of the pendulum, to the nearest centimetre?



49. Solve $\triangle ABC$ with vertices A(2, 1), B(7, 4), and C(1, 5). Round each measure to the nearest tenth of a unit.

50. Try to solve a triangle with side lengths 3 cm, 8 cm, and 4 cm using the cosine law. What do you find? Explain why.

51. Solve the following system of equations.

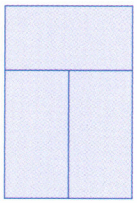
$a + b + c = 9$

$ab + bc + ac = 26$

$abc = 24$

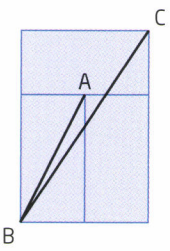
52. If x, y, and z are three different numbers whose sum is zero and whose product is two, what is the value of $x^3 + y^3 + z^3$?

53. Three identical rectangles are placed as shown to form a large rectangle that has an area of 1536 m^2.



a) Find the area of a square that has the same perimeter as the large rectangle.

b) Two diagonals are drawn, as shown. Find the measure of $\angle ABC$, to the nearest degree.



54. The area of a triangle is 24 cm^2. Its height is 2 cm more than its base. Determine the base and the height.

55. The vertices of a cube lie on a sphere of radius 5 cm. Find the volume of the cube, to the nearest tenth of a cubic centimetre.

56. The number 144 is a perfect square. The sum of the digits of 144 is 9, which is also a perfect square. How many other three-digit numbers have this property?

57. A rectangular floor is covered with identical square tiles. The floor is 40 tiles long and 30 tiles wide. If a straight line is drawn across the floor from corner to corner, how may tiles will it cross?

58. If $9x + 24 = A[x + B(x + C)]$ and A, B, and C are integers, find the values of A, B, and C.

Prerequisite Skills Appendix

Adding Polynomials

To add $2x + 5y - 4$, add the like terms.

$$\begin{array}{r} 3x - 2y - 5 \\ \hline \end{array}$$

$$\begin{array}{r} 2x + 5y - 4 \\ 3x - 2y - 5 \\ \hline 5x + 3y - 9 \end{array}$$

1. Add.

a) $\begin{array}{r} 3x + 2y + 5 \\ 4x + 3y + 7 \\ \hline \end{array}$

b) $\begin{array}{r} 3x - 4y - 5 \\ 6x - 2y - 7 \\ \hline \end{array}$

c) $\begin{array}{r} x^2 + 6x - 4 \\ 2x^2 - 4x + 8 \\ \hline \end{array}$

d) $\begin{array}{r} 3a^2 - a + 2 \\ 2a^2 + 4a - 1 \\ \hline \end{array}$

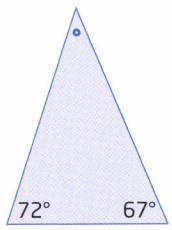
e) $\begin{array}{r} 2y^2 + y - 3 \\ 3y^2 - y + 2 \\ \hline \end{array}$

f) $\begin{array}{r} -6a + 2b + 4 \\ 4a - 3b - 7 \\ \hline \end{array}$

Angle Properties

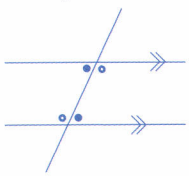
To find the measure of x, recall that the sum of the interior angles of a triangle is 180°.

$$x + 72° + 67° = 180°$$
$$x + 139° = 180°$$
$$x = 180° - 139°$$
$$x = 41°$$

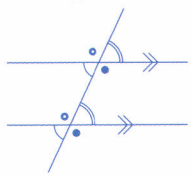


When a transversal intersects two parallel lines:

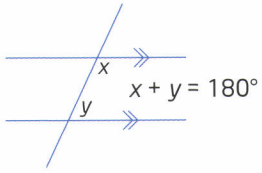
a) the alternate angles are equal



b) the corresponding angles are equal

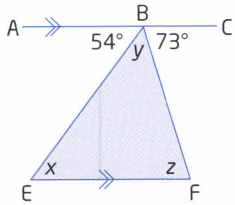


c) the co-interior angles are supplementary



To find the measures of x, y, and z, use the fact that, since AC is parallel to EF, the alternate angles are equal.

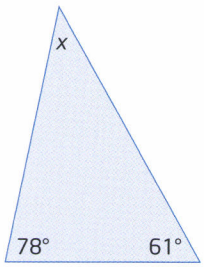
$\angle BEF = \angle ABE$ (alternate angles)
$x = 54°$
$\angle BFE = \angle CBF$ (alternate angles)
$z = 73°$



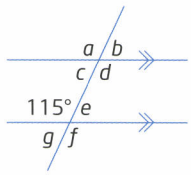
$$x + y + z = 180°$$
$$54° + y + 73° = 180°$$
$$y + 127° = 180°$$
$$y = 180° - 127°$$
$$y = 53°$$

1. Find the unknown angle measures.

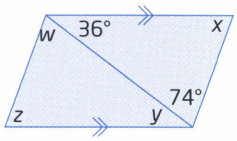
a)



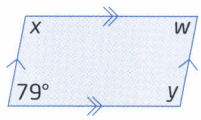
b)



c)



d)



Common Factoring

To factor the expression $10y^2 + 8y$, determine the greatest common factor of both terms. Refer to greatest common factors in this appendix.

$10y^2 = 2 \times 5 \times y \times y$
$8y = 2 \times 2 \times 2 \times y$

The greatest common factor is $2y$.

The second factor is $\frac{10y^2}{2y} + \frac{8y}{2y}$ or $5y + 4$.

The factors of $10y^2 + 8y$ are $2y$ and $5y + 4$.
Therefore, $10y^2 + 8y = 2y(5y + 4)$.

1. State the missing factor.

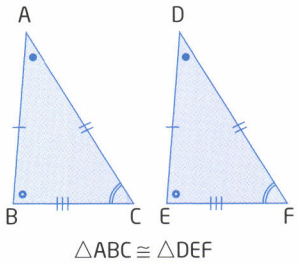
a) $6x + 8y = 2(\blacksquare)$
b) $2x^2 - 5x = x(\blacksquare)$
c) $4abc + 10ab = 2ab(\blacksquare)$
d) $8a^3 - 12a^2 = 4a^2(\blacksquare)$
e) $-5ab - 10c = -5(\blacksquare)$
f) $-4x^2 + 8x = -4x(\blacksquare)$

2. Factor.

a) $5y + 15$
b) $24x - 16$
c) $4ab + 6a$
d) $3x^2 - 18x$
e) $2x^2 + 4x - 6$
f) $6x^3 - 3x^2 + 9x$
g) $8ab^2 + 4ab + 12a^2b$
h) $10y^3 - 10$

Congruent Triangles

Congruent triangles have the same shape and the same size. When two triangles are congruent, their corresponding angles and corresponding sides are equal.



$\triangle ABC \cong \triangle DEF$

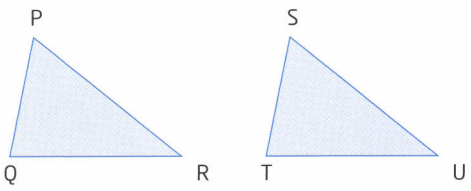
$\triangle ABC$ is congruent to $\triangle DEF$.

The following corresponding parts are equal.

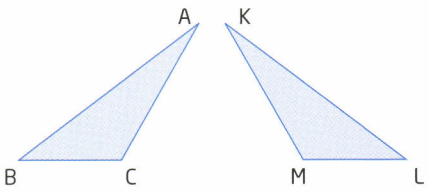
$\angle A = \angle D$ $\quad$ $AB = DE$
$\angle B = \angle E$ $\quad$ $AC = DF$
$\angle C = \angle F$ $\quad$ $BC = EF$

1. List the corresponding equal parts in each pair of congruent scalene triangles.

a)



b)



Evaluating Expressions

To evaluate the expression $4x^2 - 5y$ for $x = 2$ and $y = -3$, substitute 2 for x and -3 for y in the expression. Then, simplify using the order of operations.

$$\begin{aligned} 4x^2 - 3y &= 4(2)^2 - 5(-3) \\ &= 4(4) - 5(-3) \\ &= 16 + 15 \\ &= 31 \end{aligned}$$

1. Evaluate for $x = 3$, $y = 2$, and $z = 1$.

a) $3x + 4$

b) $2x + 4y - 3z$

c) $3(x + z)$

d) $x^2 - y^2 + z^2$

e) $3xy - yz + 2$

f) $2y^2 + x$

g) $3(4z + 3y)$

h) $2z(x - 2y)$

i) $3y^2 - 2x^2 + 3z^2$

j) $x(z - 4y)$

k) $(yz)^2$

l) $4(x - y - z)$

2. Evaluate for $x = -2$, $y = 3$, and $z = -1$.

a) $x + y - z$

b) $4x + 3y$

c) $5x + 3z - 4y$

d) $3xyz - 6$

e) $4yz - x$

f) $xy + yz - xz$

g) $x^2 + y^2 - z^2$

h) $3(2z - x)$

i) $(x + y)(y - z)$

j) $(xyz)^3$

k) $2z(4y - 3x)$

l) $2z^2 - 3y^2 - 4x^2$

To complete the table of values for $y = x^2 - 5x$, substitute the given values for x in $x^2 - 5x$ and determine y.

$y = x^2 - 5x$

x	y
-2	
0	
2	

When $x = -2$, $y = (-2)^2 - 5(-2)$
$= 4 + 10$
$= 14$

When $x = 0$, $y = (0)^2 - 5(0)$
$= 0 - 0$
$= 0$

When $x = 2$, $y = (2)^2 - 5(2)$
$= 4 - 10$
$= -6$

$y = x^2 - 5x$

x	y
-2	14
0	0
2	-6

3. Copy and complete each table of values.

a) $y = x + 4$

x	y
2	
1	
0	
-1	
-2	

b) $y = 2x - 3$

x	y
2	
1	
0	
-1	
-2	

c) $y = -x + 5$

x	y
4	
2	
0	
-2	
-4	

d) $y = x^2 + 1$

x	y
2	
1	
0	
-1	
-2	

e) $y = x^2 + 2x$

x	y
2	
1	
0	
-1	
-2	

f) $y = x^2 - 2x + 4$

x	y
4	
2	
0	
-2	
-4	

Evaluating Radicals

Since $7 \times 7 = 49$, $\sqrt{49} = 7$.

Since $0.4 \times 0.4 = 0.16$, $\sqrt{0.16} = 0.4$.

1. Evaluate.

a) $\sqrt{4}$ **b)** $\sqrt{25}$ **c)** $\sqrt{0.81}$

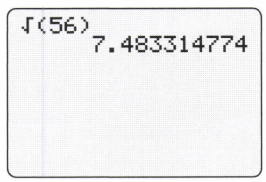
d) $\sqrt{1.21}$ **e)** $\sqrt{0.09}$ **f)** $\sqrt{0.01}$

g) $\sqrt{225}$ **h)** $\sqrt{1.69}$

To evaluate $\sqrt{56}$, to the nearest tenth, use a calculator.

$\sqrt{56} \doteq 7.483\,314\,774$

so $\sqrt{56} = 7.5$, to the nearest tenth.



2. Evaluate, to the nearest tenth.

a) $\sqrt{44}$ **b)** $\sqrt{129}$ **c)** $\sqrt{3422}$

d) $\sqrt{20.5}$ **e)** $\sqrt{89.4}$ **f)** $\sqrt{747}$

g) $\sqrt{65\,771}$ **h)** $\sqrt{0.7}$

Expanding Expressions

To expand $3x(x - 4)$, use the distributive property.

$$\begin{aligned} 3x(x - 4) &= 3x(x - 4) \\ &= 3x^2 - 12x \end{aligned}$$

1. Expand.

a) $2(x + 3)$ **b)** $3(x + y - 7)$ **c)** $5(a - b + c)$

d) $-2(5a - 4)$ **e)** $-(2x - y)$ **f)** $x(x + 6)$

g) $2x(3x + 7)$ **h)** $x(x^2 - x + 5)$ **i)** $-3a(a^2 + 2a - 1)$

Exponent Rules

To multiply powers with the same base, add the exponents.

$$\begin{aligned} x^2 \times x^3 &= x^{2+3} \\ &= x^5 \end{aligned}$$

To divide powers with the same base, subtract the exponents.

$$\begin{aligned} x^5 \div x^2 &= x^{5-2} \\ &= x^3 \end{aligned}$$

To raise a power to a power, multiply the exponents.

$$\begin{aligned} (x^2)^4 &= x^{2 \times 4} \\ &= x^8 \end{aligned}$$

1. Simplify, using the exponent rules. Express each answer in exponential form.

a) $2^2 \times 2^5$ **b)** $3^6 \times 3^4$ **c)** $4^2 \times 4^3 \times 4^2$

d) $5^3 \times 5^2 \times 5$ **e)** $2^5 \div 2^3$ **f)** $3^7 \div 3^4$

g) $4^6 \div 4$ **h)** $(2^3)^2$ **i)** $(3^4)^3$

j) $y^4 \times y^7$ **k)** $z^3 \times z^3$ **l)** $y^5 \div y^4$

m) $z^8 \div z^2$ **n)** $(x^3)^5$ **o)** $(y^2)^8$

p) $3x^4 \times 2x^3$ **q)** $(-2y^3)(-4y^4)$ **r)** $-10m^7 \div (-2m^3)$

s) $(3y^3)^2$ **t)** $(-2x^3)^3$

First Differences

First differences are calculated from tables of values in which the x-coordinates are evenly spaced. First differences are found by subtracting consecutive y-coordinates.

If the first differences are constant, the relation is linear. If the first differences are not constant, the relation is non-linear.

This relation is linear.

x	y	First Differences
1	3	
2	5	5 – 3 = 2
3	7	7 – 5 = 2
4	9	9 – 7 = 2

This relation is non-linear.

x	y	First Differences
1	1	
2	4	4 – 1 = 3
3	9	9 – 4 = 5
4	16	16 – 9 = 7

1. Use first differences to determine whether each relation is linear or non-linear.

a)

x	y
1	5
2	8
3	11
4	14

b)

x	y
1	7
2	5
3	3
4	1

c)

x	y
1	2
2	5
3	10
4	17

d)

x	y
1	2
2	6
3	10
4	14

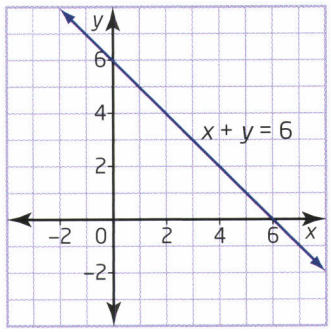
Graphing Equations

To graph the line $x + y = 6$ using a table of values, choose suitable values for x.
Complete a table of values by finding the value of y for each value of x.

$x + y = 6$

x	y
5	1
3	3
1	5
−1	7

Plot the points on a grid and draw a line through the points.



1. Graph each equation using a table of values.

a) $x + y = 4$ **b)** $x - y = 2$

c) $y = x + 2$ **d)** $y = 2x + 1$

To graph $x + 2y = 6$ using the intercepts, find the points where the graph of $x + 2y = 6$ crosses the x- and y-axes.

To find the x-intercept, let $y = 0$.

$x + 2y = 6$

$x + 2(0) = 6$

$x = 6$

One point on the line is (6, 0).

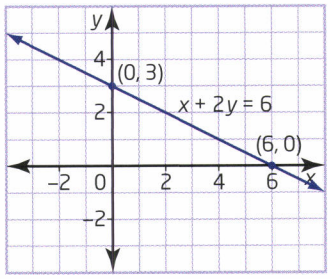
To find the y-intercept, let $x = 0$.

$x + 2y = 6$

$0 + 2y = 6$

$y = 3$

Another point on the line is (0, 3).



Plot the points on a grid.
Draw a line through the points.

2. Graph each equation using the intercepts.

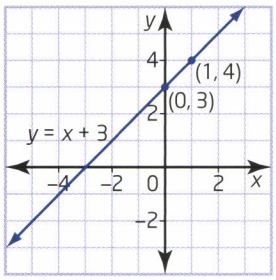
a) $x + y = 3$ **b)** $x - y = 4$

c) $4x + y = 8$ **d)** $2x - 5y = 10$

To graph $y = x + 3$ using the slope and y-intercept, first find the point where the graph crosses the y-axis.

Since the y-intercept is 3, plot the point (0, 3). The slope is 1. Use the slope to find another point on the line.

Draw the graph.



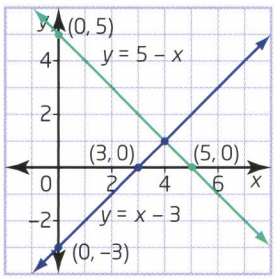
3. Graph each equation using the slope and y-intercept.

a) $y = x + 3$ **b)** $y = -x - 4$

c) $y = 2x + 3$ **d)** $y = 3x - 1$

To find the point of intersection of the lines $y = x - 3$ and $y = 5 - x$, graph the lines on the same set of axes.

The coordinates of the point of intersection are (4, 1).



4. Graph each pair of lines and find the coordinates of the point of intersection.

a) $y = x - 4$ and $y = 8 - x$ **b)** $y = x + 3$ and $y = 7 - x$

c) $y = x - 6$ and $y = 2 - x$ **d)** $y = x + 5$ and $y = 3 - x$

Greatest Common Factors

To determine the greatest common factor (GCF) of $4c^2d$ and $6cd^3$, write each expression as a product. Then, write the factors that are common to both.

$4c^2d = 2 \times 2 \times c \times c \times d$

$6cd^3 = 2 \times 3 \times c \times d \times d \times d$

The GCF of $4c^2d$ and $6cd^3$ is $2 \times c \times d$ or $2cd$.

1. Determine the GCF of each pair.

a) $4x$, $10x$ **b)** $12y$, $8y$ **c)** $15z$, $10z$

d) $30a$, $20a$ **e)** $6x^2$, $14x$ **f)** $14ab$, $7abc$

g) $12x^3$, $18x^2$ **h)** $9ab^2c$, $7a^2bc$

2. Determine the GCF of each set.

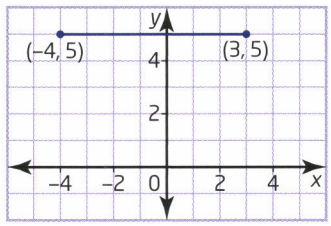
a) $4ab$, $2abc$, $6a$ **b)** $3xyz$, $9yz$, $12xy$

c) $8x^3$, $20x^2$, $24x^4$ **d)** $15mn^2$, $18m^2n$, $12mn$

e) $10rt^3$, $15r^2t^2$, $20r^3t^4$ **f)** $9rs$, $18ab$, $27xy$

Lengths of Line Segments

The length of any line segment is a positive number. To find the length of the horizontal line segment joining (3, 5) and (−4, 5), subtract the lesser x-coordinate from the greater x-coordinate.



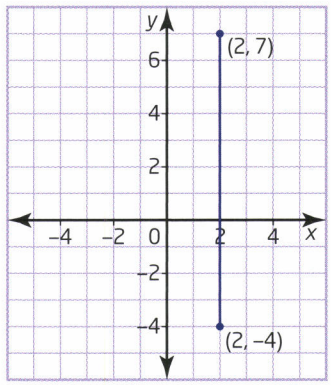
$$\begin{aligned} 3 - (-4) &= 3 + 4 \\ &= 7 \end{aligned}$$

The length of the horizontal line segment joining (3, 5) and (−4, 5) is 7 units.

To find the length of the vertical line segment joining (2, −4) and (2, 7), subtract the y-coordinates.

$$\begin{aligned} 7 - (-4) &= 7 + 4 \\ &= 11 \end{aligned}$$

The length of the vertical line segment joining (2, −4) and (2, 7) is 11 units.



1. Find the length of the line segment joining each pair of points.

a) (8, 7) and (2, 7)

b) (5, 1) and (9, 1)

c) (3, 8) and (3, 2)

d) (5, 1) and (5, 9)

e) (2, 4) and (−5, 4)

f) (1, 7) and (1, −6)

g) (−4, −3) and (−8, −3)

h) (−2, 5) and (−2, −7)

i) (6, 3) and (0, 3)

j) (−6, −7) and (−6, 0)

k) (−6, 4) and (8, 4)

l) (−7, −9) and (−7, −3)

Like Terms

Like terms have exactly the same variables raised to exactly the same exponents. An expression is in simplest form when there are no like terms.

To simplify $x^2 + 5x - 7 - 2x + 4 + 3x^2$, collect like terms.

$$\begin{aligned} x^2 + 5x - 7 - 2x + 4 + 3x^2 &= x^2 + 3x^2 + 5x - 2x - 7 + 4 \\ &= 4x^2 + 3x - 3 \end{aligned}$$

1. Simplify by collecting like terms.

a) $4x + 7x - 5x$

b) $2y - 6 + 3y - 8$

c) $2x + 3y - 2 + x - 4y - 5$

d) $3a - 2b + 4c - 3b + a$

e) $x^2 - 2x - 7x - 3 - 3x^2$

f) $3 - t^2 + 8t - 7 - 6t - t$

g) $3x + 2y - 5 - 3 + x + 7y$

h) $3y^2 - 4y^2 - 11y + 3 - 7y$

i) $15 - 4t - 5t - t^2 - t^2 - t$

Number Skills

To evaluate expressions, use the order of operations (BEDMAS).

$$\begin{aligned}
&(-2)(-6) - 5(-3)^2 \\
&= 12 - 5(9) \\
&= 12 - 45 \\
&= -33
\end{aligned}$$

$$\begin{aligned}
\frac{2}{3} + \frac{1}{2}\left(2\frac{3}{4} - 1\frac{1}{2}\right) &= \frac{2}{3} + \frac{1}{2}\left(\frac{11}{4} - \frac{3}{2}\right) \\
&= \frac{2}{3} + \frac{1}{2}\left(\frac{11}{4} - \frac{6}{4}\right) \\
&= \frac{2}{3} + \frac{1}{2}\left(\frac{5}{4}\right) \\
&= \frac{2}{3} + \frac{5}{8} \\
&= \frac{16}{24} + \frac{15}{24} \\
&= \frac{31}{24} \text{ or } 1\frac{7}{24}
\end{aligned}$$

1. Evaluate each expression.

a) $-3(5) + (-6)(-8)$

b) $(12)(11) - (-9)(7)$

c) $-10(3)(2) + (-12)(7)(-2)$

d) $\frac{3}{5} + 2\frac{2}{3}$

e) $8 - 2\left(\frac{7}{8} + 3\frac{3}{4}\right)$

f) $\left(\frac{3}{4}\right)^2\left(\frac{8}{3}\right)$

g) $\frac{5}{2} - \frac{4}{5}\left(\frac{3}{2}\right)^3$

h) $3.2\left(\frac{1}{2}\right) - 2.5\left(\frac{3}{5}\right) + 1.4\left(\frac{2}{7}\right)$

i) $0.3(5.5)^2 - (-6.7)(2.1)^3 + 4.2(-1.1)^5$

2. Order each set of fractions from least to greatest.

a) $\frac{5}{8}, \frac{2}{3}, \frac{3}{5}, \frac{7}{12}, \frac{1}{2}$

b) $3\frac{3}{4}, 3\frac{7}{8}, 3\frac{5}{9}, 3\frac{6}{7}$

3. Explain why the two expressions are not equal.

a) $\sqrt{9 + 16}, \sqrt{9} + \sqrt{16}$

b) $(x + y)^2, x^2 + y^2$

c) $\frac{2}{3} + \frac{5}{6}, \frac{7}{9}$

Percents

The table shows how equivalent fractions, percents, and decimals can be expressed.

Fraction	Percent	Decimal
$\frac{63}{100}$	63%	0.63
$\frac{8}{100} = \frac{2}{25}$	8%	0.08
$\frac{5}{1000} = \frac{1}{200}$	0.5%	0.005
$\frac{150}{100} = \frac{3}{2}$	150%	1.5

1. Copy and complete the following table. Express all fractions in lowest terms.

	Fraction	Percent	Decimal
a)	$\frac{75}{100}$		
b)	$\frac{1}{2}$		
c)	$8\frac{2}{5}$		
d)		34%	
e)		0.03%	
f)		5.6%	
g)			0.45
h)			0.03
i)			2.68

Polynomials

The degree of a polynomial in one variable is the greatest power of the variable in any term.

For the polynomial $3x^2 - 2x + 7$, the greatest power, 2, is contained in the term $3x^2$.

$3x^2 - 2x + 7$ is a second-degree polynomial.

1. State the degree of each polynomial.

a) $4x - 3$

b) x^3

c) $4 - 2y^2 - 5y$

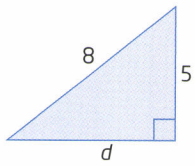
d) $6m^2 - m^3 - 2$

e) $2x + 7 - 5x^4$

f) $2 - 4t^5 - 3t^3 + 9t$

Pythagorean Theorem

To find the length of d in the right triangle, to the nearest tenth of a unit, use the Pythagorean theorem. This states that, in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



$d^2 + 5^2 = 8^2$

$d^2 + 25 = 64$

$d^2 = 64 - 25$

$d^2 = 39$

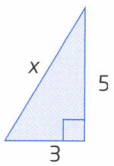
$d = \sqrt{39}$

$d \doteq 6.244\ 997\ 998$

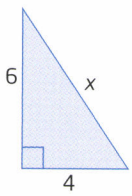
The length of d is 6.2 units, to the nearest tenth of a unit.

1. In each right triangle, find the unknown side length, to the nearest tenth of a unit.

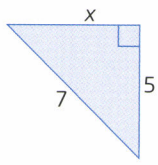
a)



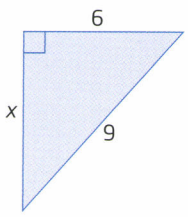
b)



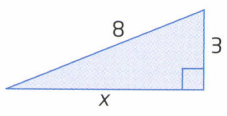
c)



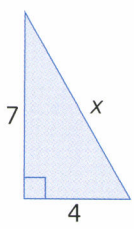
d)



e)



f)



Simplifying Expressions

To simplify $3(x + 2) - (x - 4)$, remove brackets and collect like terms.

$$\begin{aligned} 3(x + 2) - (x - 4) &= 3(x + 2) - 1(x - 4) \\ &= 3x + 6 - x + 4 \\ &= 2x + 10 \end{aligned}$$

1. Simplify.

a) $5x + 2(x + 7)$

b) $3(2a - 7) + 3a$

c) $4(x - 3) - 2x$

d) $-5(y - 3) + 6$

e) $2(t + 4) + 3(t - 4)$

f) $8(y - 3) - (y + 6)$

g) $-4(z + 3) - 2(z - 2)$

h) $7(2 - w) - (w - 3)$

i) $6(x - 4) + 2(3 + x)$

2. Simplify.

a) $(3x + 4y) + (2x + 3y)$

b) $(4r - 3s) + (r + 4s)$

c) $(3p + 5q) - (4p - q)$

d) $2(x - 3y) + 3(6x - y)$

e) $4(3a + 5b) - (7a - b)$

f) $-4(c - 5d) + 3c - d$

g) $4a - b + 3c - 2(a + b - c)$

h) $3(x + y - 2z) + 7x - 5$

i) $6x + 4(2x + 3y - z) + 5y$

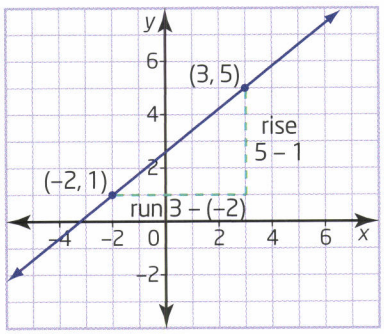
Slope

To find the slope of the line passing through the points $(-2, 1)$ and $(3, 5)$, use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - 1}{3 - (-2)}$$

$$= \frac{4}{5}$$



1. Find the slope of the line passing through each pair of points.

a) $(0, 0)$ and $(2, 6)$

b) $(0, 0)$ and $(4, 2)$

c) $(1, 2)$ and $(3, 6)$

d) $(2, 5)$ and $(3, 7)$

e) $(3, 0)$ and $(0, 6)$

f) $(-3, 5)$ and $(2, 6)$

g) $(-4, 2)$ and $(5, 2)$

h) $(-3, -4)$ and $(-1, -8)$

i) $(5, -7)$ and $(3, -10)$

Parallel lines have the same slope. The slope of a line parallel to $y = 2x + 3$ is 2.

The product of the slopes of perpendicular lines is -1.

The slope of a line perpendicular to $y = 2x + 3$ is $-\frac{1}{2}$, since $2 \times -\frac{1}{2} = -1$.

2. State the slope of a line parallel to and a line perpendicular to each line.

a) $y = 3x + 5$

b) $y = -2x - 4$

c) $y = -x + 7$

d) $y = \frac{1}{4}x - 4$

e) $y = -\frac{2}{3}x + 1$

f) $y = \frac{4}{5}x - 6$

Solving Equations

To solve $2x - 3 = 11$, isolate the variable.

$$2x - 3 = 11$$

Add 3 to both sides: $2x - 3 + 3 = 11 + 3$

Simplify: $2x = 14$

Divide both sides by 2: $\frac{2x}{2} = \frac{14}{2}$

Simplify: $x = 7$

To check, substitute 7 for x in the original equation.

L.S. $= 2x - 3$ **R.S.** $= 11$

$= 2(7) - 3$

$= 14 - 3$

$= 11$

Since **L.S.** = **R.S.**, the solution is $x = 7$.

1. Solve and check.

a) $3x + 2 = 11$

b) $2x - 5 = 7$

c) $5x - 2 = -17$

d) $4x - 7 = 9$

e) $1 - 2x = 15$

f) $8 + 7x = -6$

g) $3x + 4 = 2x + 5$

h) $5x + 7 = 3x - 9$

i) $2x - 3 = 5x + 9$

To solve $3(x - 2) = 9$, expand to remove the brackets.

$$3(x - 2) = 9$$

Expand: $3x - 6 = 9$

Add 6 to both sides: $3x - 6 + 6 = 9 + 6$

Simplify: $3x = 15$

Divide both sides by 3: $\frac{3x}{3} = \frac{15}{3}$

Simplify: $x = 5$

To check, substitute 5 for x in the original equation.

L.S. $= 3(x - 2)$ **R.S.** $= 9$

$= 3(5 - 2)$

$= 3(3)$

$= 9$

Since **L.S.** = **R.S.**, the solution is $x = 5$.

2. Solve and check.

a) $2(x + 4) = 10$

b) $4(x - 1) = 16$

c) $6(x - 5) + 7 = 1$

d) $5(x + 1) = -15$

e) $2(x - 3) = 3(x + 1)$

f) $5(x - 3) = 3(x + 7)$

g) $7(x + 2) - 3(x - 4) = 30$

h) $5(x - 2) + 3(x - 1) = 3$

i) $7(x + 6) - (x + 7) = -1$

To solve $\frac{x-1}{3} = \frac{2x-3}{5}$, eliminate fractions by multiplying by the lowest common denominator, 15.

$$\frac{x-1}{3} = \frac{2x-3}{5}$$

Multiply both sides by 15: $15 \times \frac{x-1}{3} = 15 \times \frac{2x-3}{5}$

Simplify: $5(x-1) = 3(2x-3)$

Expand: $5x - 5 = 6x - 9$

Add 5 to both sides: $5x - 5 + 5 = 6x - 9 + 5$

Simplify: $5x = 6x - 4$

Subtract $6x$ from both sides: $5x - 6x = 6x - 4 - 6x$

Simplify: $-x = -4$

Divide both sides by –1: $x = 4$

To check, substitute 4 for x in the original equation.

L.S. $= \frac{x-1}{3}$

$= \frac{4-1}{3}$

$= \frac{3}{3}$

$= 1$

R.S. $= \frac{2x-3}{5}$

$= \frac{2(4)-3}{5}$

$= \frac{5}{5}$

$= 1$

Since **L.S.** = **R.S.**, the solution is $x = 4$.

3. Solve and check.

a) $\frac{x}{3} + 1 = 4$

b) $\frac{x}{2} + \frac{1}{3} = \frac{5}{6}$

c) $\frac{x}{5} - \frac{1}{2} = \frac{3}{10}$

d) $\frac{x+3}{2} = x - 2$

e) $\frac{x-1}{2} = \frac{x+1}{3}$

f) $\frac{2x+1}{3} = \frac{x-6}{8}$

g) $\frac{x+4}{2} - \frac{x+10}{4} = -1$

h) $\frac{x+7}{5} - \frac{2x}{3} = 0$

i) $\frac{x+1}{8} + \frac{x+3}{5} = 3$

Solving Proportions

Proportions can be solved using the cross-product rule, which states that, if $\frac{a}{b} = \frac{c}{d}$, then $a \times d = b \times c$.

To solve $2:5 = 3:x$, first write the proportion in fraction form, $\frac{2}{5} = \frac{3}{x}$.

Then, use the cross-product rule.

$$\frac{2}{5} = \frac{3}{x}$$
$$2x = 5 \times 3$$
$$2x = 15$$
$$x = \frac{15}{2} \text{ or } 7.5$$

1. Solve for x. Express each answer as a fraction in lowest terms.

a) $\frac{x}{6} = \frac{2}{5}$

b) $\frac{4}{5} = \frac{x}{3}$

c) $\frac{2}{x} = \frac{3}{7}$

d) $\frac{3}{2} = \frac{4}{x}$

e) $x:4 = 5:6$

f) $8:x = 6:7$

g) $4:9 = x:6$

h) $5:8 = 2:x$

2. Solve for x. Express each answer as a decimal. Round to the nearest hundredth, if necessary.

a) $\frac{x}{3.4} = 4.5$

b) $\frac{2.6}{x} = 5.8$

c) $3.2 = \frac{x}{4.3}$

d) $\frac{x}{1.5} = \frac{2.6}{3}$

e) $\frac{0.8}{x} = \frac{4.2}{1.4}$

f) $\frac{3.8}{1.7} = \frac{2.8}{x}$

g) $6.3:x = 7.5:8.3$

h) $1.4:0.5 = 2.9:x$

Subtracting Polynomials

To subtract $\begin{array}{r} 5x - 2y + 8 \\ 2x + 3y - 2 \end{array}$

add the opposite of the polynomial that is being subtracted.
The opposite of $2x + 3y - 2$ is $-2x - 3y + 2$.

$$\begin{array}{r} 5x - 2y + 8 \\ -2x - 3y + 2 \\ \hline 3x - 5y + 10 \end{array}$$

1. Subtract.

a) $\begin{array}{r} 5x + 6y + 8 \\ 3x + 2y + 5 \end{array}$

b) $\begin{array}{r} 7x - 5y - 8 \\ 4x - 3y - 7 \end{array}$

c) $\begin{array}{r} 3x^2 - 6x - 4 \\ 4x^2 + 2x + 5 \end{array}$

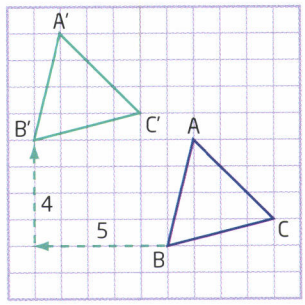
d) $\begin{array}{r} 5a^2 - a + 3 \\ 2a^2 - 7a - 8 \end{array}$

e) $\begin{array}{r} -4a + 3b + 1 \\ 3a - 3b - 6 \end{array}$

f) $\begin{array}{r} 4y^2 + 2y - 3 \\ -3y^2 + 6y - 1 \end{array}$

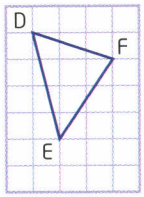
Transformations

To translate $\triangle ABC$ 5 units to the left and 4 units up, translate each vertex of the triangle 5 units to the left and 4 units up. Join the new points to form the translation image, $\triangle A'B'C'$. $\triangle ABC$ and its translation image, $\triangle A'B'C'$, are congruent, since corresponding side lengths and angles are equal.



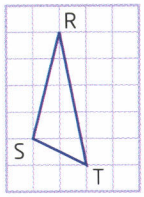
1. Draw each triangle on grid paper. Then, draw the translation image for the given translation.

a)



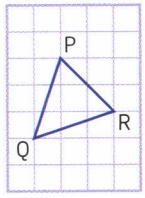
3 right, 3 down

b)



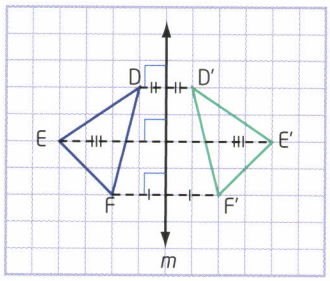
2 left, 4 down

c)



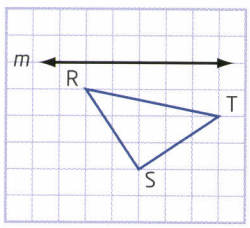
2 right, 4 up

To reflect $\triangle DEF$ in the reflection line m, reflect each vertex of $\triangle DEF$ in the line.
Each vertex of $\triangle DEF$ and its reflection image are the same perpendicular distance from the line m.
Join the new points to form the reflection image, $\triangle D'E'F'$.
$\triangle DEF$ and its reflection image, $\triangle D'E'F'$, are congruent, since corresponding side lengths and angles are equal.

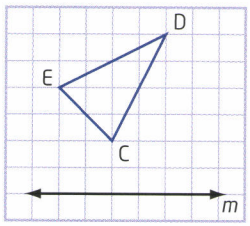


2. Draw each triangle on grid paper. Then, draw the reflection image in the given reflection line.

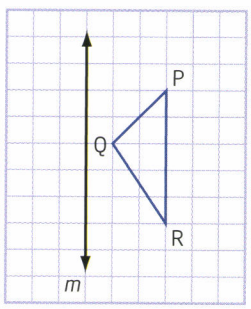
a)



b)



c)



Technology Appendix

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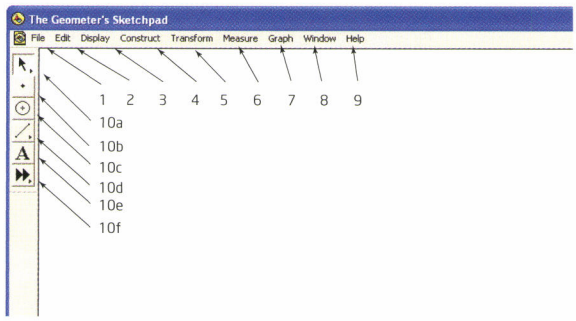
The Geometer's Sketchpad® Basics

Menu Bar



1 **File** menu—open/save/print sketches
2 **Edit** menu—undo/redo actions/set preferences
3 **Display** menu—control appearance of objects in sketch
4 **Construct** menu—construct new geometric objects based on objects in sketch
5 **Transform** menu—apply geometric transformations to selected objects
6 **Measure** menu—make various measurements on objects in sketch
7 **Graph** menu—create axes and plot measurements and points
8 **Window** menu—manipulate windows
9 **Help** menu—access the help system, an excellent reference guide
10 **Toolbox**—access tools for creating, marking, and transforming points, circles, and straight objects (segments, lines, and rays); also includes text and information tools
10a **Selection Arrow Tool** (Arrow)—select and transform objects
10b **Point Tool** (Dot)—draw points
10c **Compass Tool** (Circle)—draw circles
10d **Straightedge Tool**—draw line segments, rays, and lines
10e **Text Tool** (Letter A)—label points and write text
10f **Custom Tool** (Double Arrow)—create or use special "custom" tools

Creating a Sketch

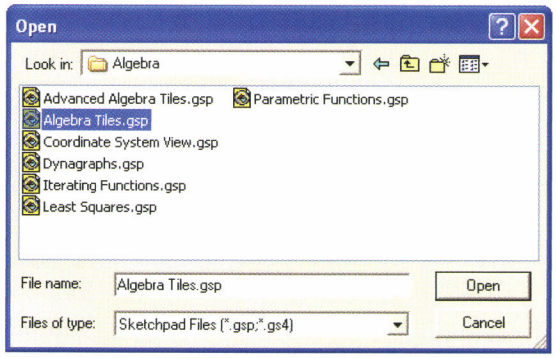
- Under the **File** menu, choose **New Sketch** to start with a new work area.

Opening an Existing Sketch

- Under the **File** menu, choose **Open...**. The **Open** dialogue box will appear.
- Choose the sketch you wish to work on. Then, click on **Open**.

OR

- Type in the name of the sketch in the **File name:** entry box. Then, click on **Open**.



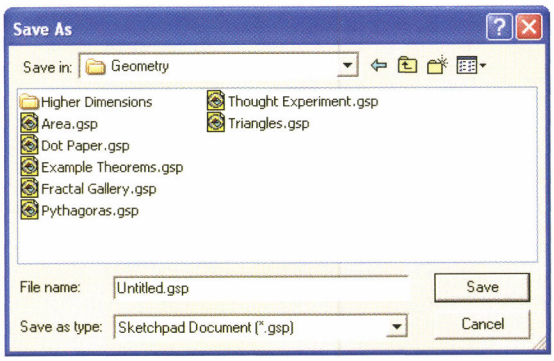
Saving a Sketch

If you are saving for the first time in a new sketch:

- Under the **File** menu, choose **Save**. The **Save As** dialogue box will appear.
- You can save the sketch with the name assigned by *The Geometer's Sketchpad*®. Click on **Save**.

OR

- Press the Backspace or Delete key to clear the name.
- Type in whatever you wish to name the sketch file. Click on **Save**.



If you have already given your file a name:

- Choose **Save** from under the **File** menu.

Closing a Sketch Without Exiting *The Geometer's Sketchpad*®

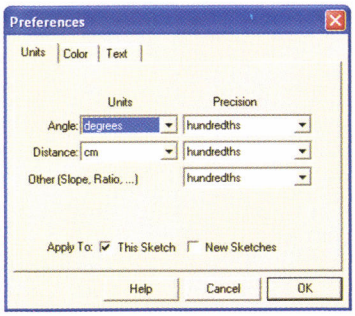
- Under the **File** menu, choose **Close**.

Exiting *The Geometer's Sketchpad*®

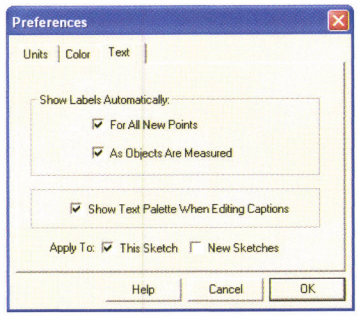
- Under the **File** menu, choose **Exit**.

Setting Preferences

- From the **Edit** menu, choose **Preferences...**.
- Click on the **Units** tab.
- Set the units and precision for angles, distances, and calculated values such as slopes or ratios.



- Click on the **Text** tab.
- If you check the auto-label box **For All New Points**, then *The Geometer's Sketchpad*® will label points as you create them.
- If you check the auto-label box **As Objects Are Measured**, then *The Geometer's Sketchpad*® will label any measurements that you define.



You can also choose whether the auto-labelling functions will apply only to the current sketch, or also to any new sketches that you create.

Be sure to click on **OK** to apply your preferences.

Selecting Points and Objects

- Choose the **Selection Arrow Tool**. The mouse cursor appears as an arrow.

To select a single point:
- Select the point by moving the cursor to the point and clicking on it.

The selected point will now appear as a darker point, similar to a *bull's-eye* ⊙.

To select an object such as a line segment or a circle:
- Move the cursor to a point on the object until it becomes a horizontal arrow.
- Click on the object. The object will change appearance to show it is selected.

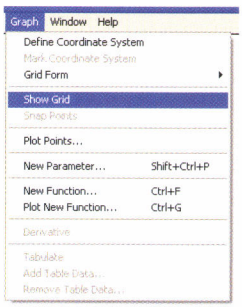
To select a number of points or objects:
- Select each object in turn by moving the cursor to the object and clicking on it.

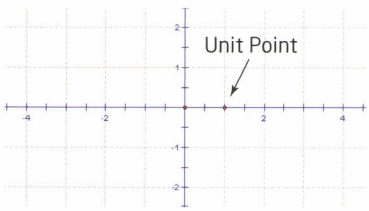
To deselect a point or an object:
- Move the cursor over it, and then click the left mouse button.
- To deselect all selected objects, click in an open area of the workspace.

Using a Coordinate System and Axes

- From the **Graph** menu, choose **Show Grid**.

The default coordinate system has an origin point in the centre of your screen and a unit point at (1, 0). Drag the origin to relocate the coordinate system and drag the unit point to change the scale.





Plotting Points

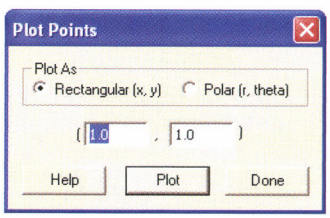
- From the **Graph** menu, choose **Show Grid**.
- If you want points plotted exactly at grid intersections, also choose **Snap Points**.
- Choose the **Point Tool**.

If you have chosen **Snap Points,** a point will "snap" to the nearest grid intersection as you move the cursor over the grid.

- Click the left mouse button to plot the point.

Alternatively, you can plot points by typing in the desired coordinates.

- From the **Graph** menu, choose **Plot Points...**. A dialogue box will appear. Type the desired x- and y-coordinates in the boxes. Then, click on **Plot**.
- When you are finished plotting points, click on **Done**.



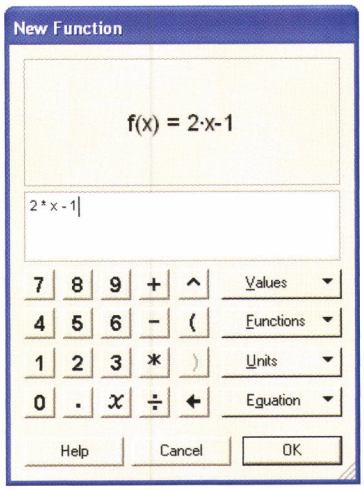
Graphing Relations

Consider the equations $y = 2x - 1$ and $y = -3x + 2$ as examples.

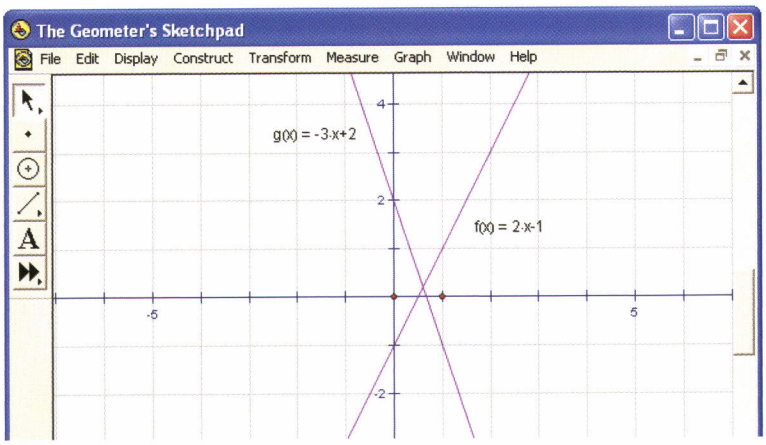
- From the **Graph** menu, choose **Show Grid**.
- From the **Graph** menu, choose **Plot New Function...**.

The calculator interface will appear. Enter the first equation:
$2 * x - 1$

- Click on **OK**. The graph of the first equation appears, along with the equation in function notation. You can move the equation next to the line.



Use the same procedure to graph the second equation.

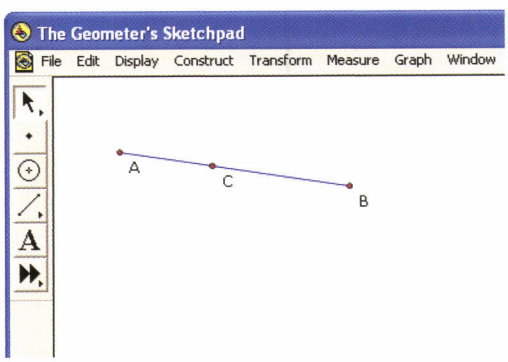


Constructing a Line Segment and a Point on the Segment

To draw a line segment:

- Use the **Point Tool**. Create two points in the workspace.
- Use the **Selection Arrow Tool** to select both points.
- From the **Construct** menu, choose **Segment**.

Your final screen will appear as shown, perhaps with different names for the points.



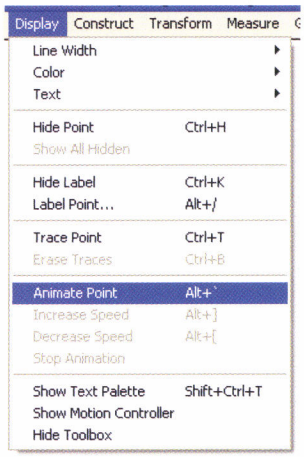
You can also use the **Straightedge Tool**. Choose this tool.

Move the cursor to the workspace. Then, click and hold the left mouse button. Drag the cursor to the desired location, and release the mouse button.

To draw a point on the line segment:
Use the **Point Tool**. Move the cursor over the line segment. The segment will change colour. Click the left mouse button. The point is plotted on the line segment. Try to drag the point. Notice that it cannot be moved off the line segment.

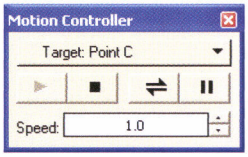
Animating a Point

To animate a point on a line segment:
- Use the **Selection Arrow Tool** to select the point on the line.
- From the **Display** menu, choose **Animate Point**.



Notice that the point moves back and forth on the line segment.

The **Motion Controller** also appears. Experiment with the controls to see what they do.

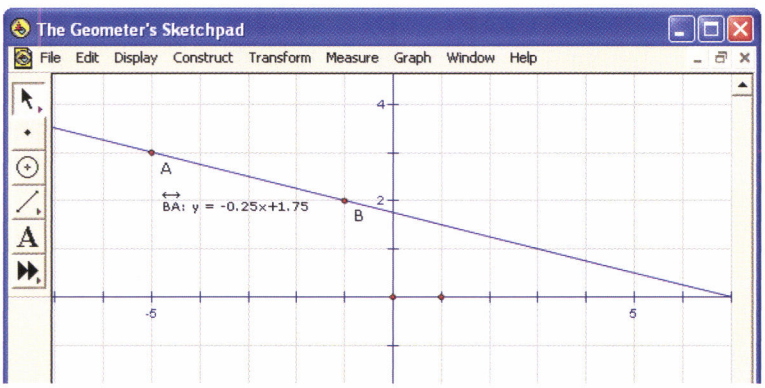


To stop the animation:
- From the **Display** menu, choose **Stop Animation**.

Constructing a Line Through Two Points and Displaying the Equation

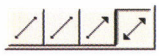
- From the **Graph** menu, choose **Show Grid**.
- Use the **Point Tool**. Plot two points.
- Ensure that both points are selected. From the **Construct** menu, choose **Line**.
- Ensure that the line is selected. From the **Measure** menu, choose **Equation**.

You can move the equation next to the line.



OR

- Click and hold the small arrowhead on the **Straightedge Tool**.
- Choose the **Line Tool**. The **Line Tool** has arrowheads at both ends.
- Move the cursor to the workspace.
- Click and drag to draw the line. Release the mouse button when the line is in the desired location. Label the line with its equation.

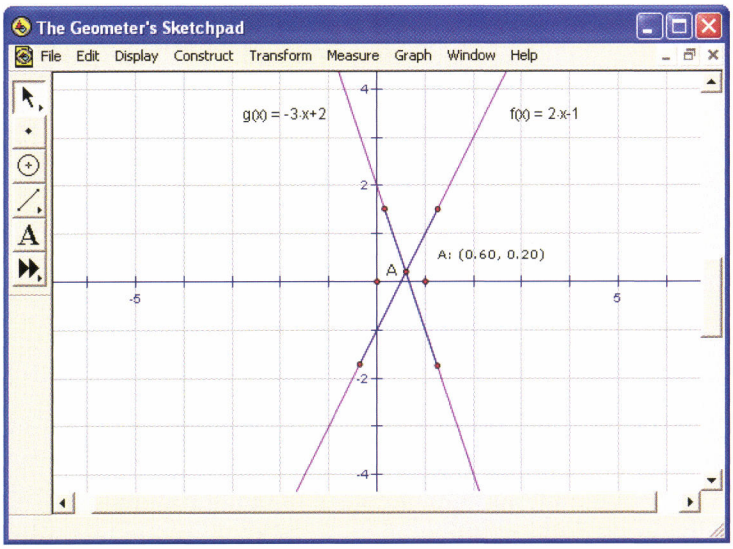


Finding a Point of Intersection

Consider the equations $y = 2x - 1$ and $y = -3x + 2$ as examples.

- Turn on the grid, and plot the graphs of the two equations.
- Use the **Point Tool** to plot two points on each line, such that the intersection lies between the points.
- Select one pair of points on a line. From the **Construct** menu, choose **Segment**. Use the same procedure to construct a segment on the other line.
- Select the two segments. From the **Construct** menu, choose **Intersection**. The point of intersection will appear.
- Select the point of intersection. From the **Measure** menu, choose **Coordinates**.

The coordinates of the point of intersection will appear.



Hiding Objects

Open a new sketch. Draw several objects such as points and line segments.

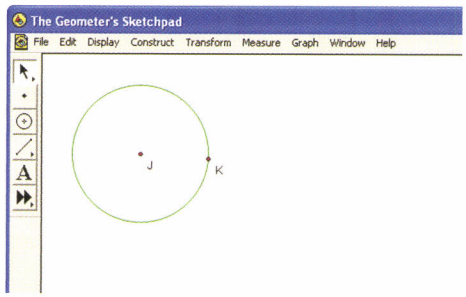
To hide a point:
- Select the point.
- From the **Display** menu, choose **Hide Point**.

To hide an object:
- Select another point and a line segment.
- From the **Display** menu, choose **Hide Objects**.

Shortcut: You can hide any selected objects by holding down CTRL, and typing **H**.
You can make hidden objects reappear by choosing **Show All Hidden** from the **Display** menu.

Constructing a Circle Using the Compass Tool

- Choose the **Compass Tool**.
- Move the cursor to the point where you want the centre of the circle.
- Click and hold the left mouse button. Drag the cursor to the desired radius.
- Release the mouse button.



Constructing a Circle by Centre and Point or Centre and Radius

To construct a circle using **Center+Point**:

- Use the **Point Tool**. Plot a point for the centre of the desired circle. Plot another point to lie on the circumference.
- Select the centre and then the point on the circumference.
- From the **Construct** menu, choose **Circle by Center+Point**.

The circle will appear.

To construct a circle using **Center+Radius**:

- Plot a point for the centre.
- Draw a line segment whose length is the desired radius.
- Select the centre and then the segment.
- From the **Construct** menu, choose **Circle by Center+Radius**.

The circle will appear.

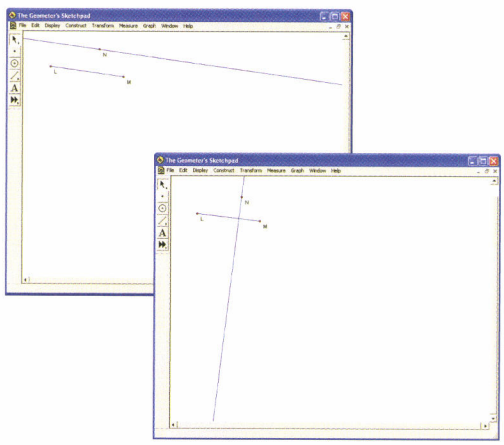
Constructing Parallel and Perpendicular Lines

To construct a line parallel to LM, passing through N:

- Select line segment LM (but not the endpoints) and point N.
- From the **Construct** menu, choose **Parallel Line**.

To construct a line perpendicular to LM, passing through N:

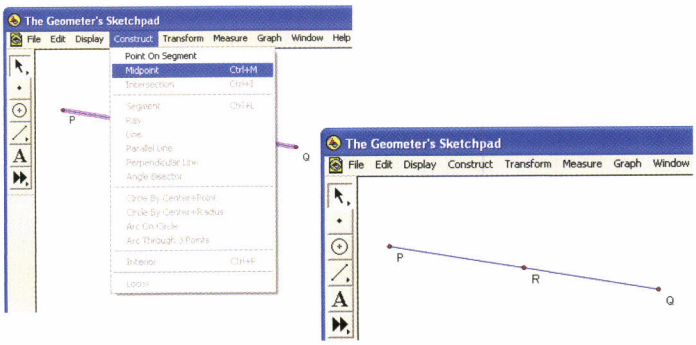
- Select line segment LM (but not the endpoints) and point N.
- From the **Construct menu**, choose **Perpendicular Line**.



Constructing Midpoints

To construct the midpoint of line segment PQ:

- Select line segment PQ (but not the endpoints).
- From the **Construct** menu, choose **Midpoint**.

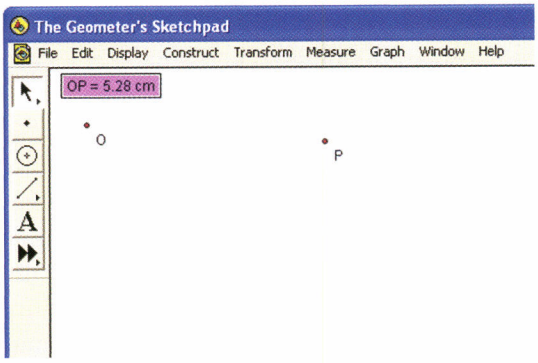


Using the Measure Menu

To measure the distance between two points:

- Ensure that nothing is selected.
- Select the two points.
- From the **Measure** menu, choose **Distance**.

The Geometer's Sketchpad® will display the distance between the points, using the units and accuracy selected in **Preferences...** under the **Edit** menu.

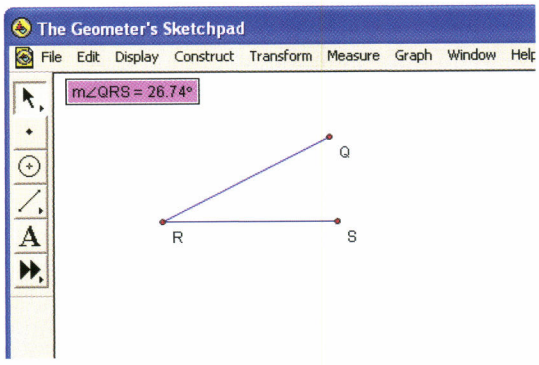


To measure the length of a line segment:

- Ensure that nothing is selected.
- Select the line segment (but not the endpoints).
- From the **Measure** menu, choose **Length**.

To measure an angle:

- Ensure that nothing is selected.
- Select the three points that define the angle so that the second point selected is the vertex of the angle. For this angle, select Q, then R, then S.
- From the **Measure** menu, choose **Angle**.

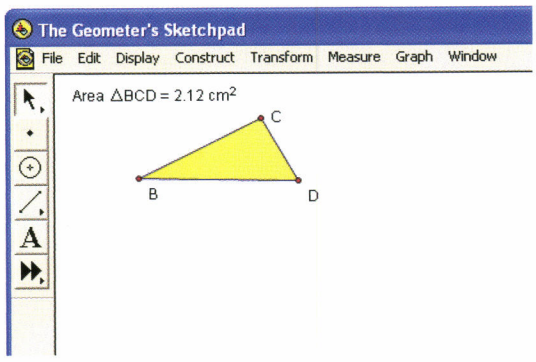


To calculate the ratio of two lengths:

- Select the two lengths to be compared.
- From the **Measure** menu, choose **Ratio**.

To measure the area of a triangle or other closed figure:

- Use the **Straightedge Tool** to draw a triangle.
- Select the points on the vertices of the triangle.
- From the **Construct** menu, choose **Triangle Interior**.
- Select the triangle interior.
- From the **Measure** menu, choose **Area**.



You can construct and measure the area of other closed figures in a similar manner.

To measure the slope of a line, ray, or line segment:

- Select the line, ray, or line segment.
- From the **Measure** menu, choose **Slope**.

Changing Labels of Measures

- Right click on the measure and choose **Label Measurement** (or **Label Distance Measurement** depending on the type of measure) from the drop-down menu.
- Type in the new label.
- Click on **OK**.

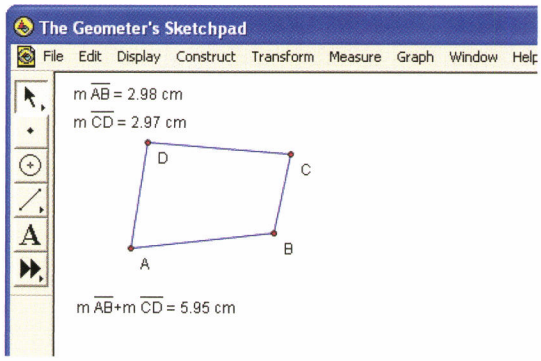
Using the On-Screen Calculator

You can use the on-screen calculator to do calculations involving measurements, constants, functions, or other mathematical operations.

To add two lengths:

- From the **Measure** menu, choose **Calculate**. The on-screen calculator will appear.
- On the workspace, select the first measure.
- On the keyboard, click on +.
- On the workspace, select the second measure.
- Click on **OK**.

The sum of the measures will appear in the workspace.

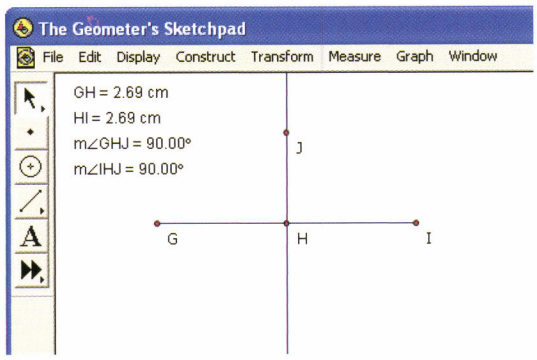


OR

- Select the two measures. Then, choose **Calculate** from the **Measure** menu. This adds the measures to the drop-down list available by clicking on the **Values** button of the on-screen calculator.
- Click on the **Values** button. Select the first measure.
- Click on +.
- Click on the **Values** button. Select the second measure.
- Click on **OK**.

Constructing the Right Bisector of a Line Segment

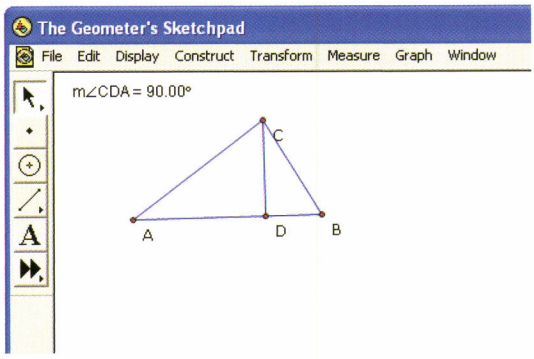
The right bisector of a line segment is a line that intersects a line segment at its midpoint at a 90° angle.



- Use the **Straightedge Tool** to draw a line segment.
- Select the line segment.
- From the **Construct** menu, choose **Midpoint**.
- Select the midpoint and the line segment.
- From the **Construct** menu, choose **Perpendicular Line**.

You can verify the construction:
- Draw a point on the perpendicular line.
- Measure the lengths of the two halves of the line segment.
- Measure the angles formed by the line segment and the perpendicular line.

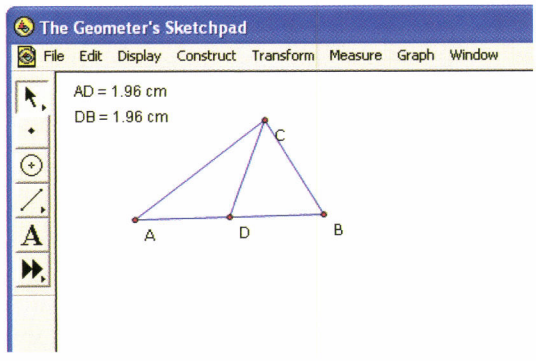
Constructing an Altitude of a Triangle



- Use the **Straightedge Tool** to construct a triangle, △ABC.
- Select one vertex, C, and the side opposite that vertex, AB.
- From the **Construct** menu, choose **Perpendicular Line**.
- Select the perpendicular line and the side BC.
- From the **Construct** menu, choose **Intersection**. Point D on side AB will appear.
- Select the perpendicular line, and choose **Hide Perpendicular Line** from the **Display** menu.
- Construct the line segment CD. CD is an altitude of △ABC.

You can check that CD is an altitude by measuring ∠CDA.

Constructing a Median of a Triangle



- Use the **Straightedge Tool** to construct a triangle, △ABC.
- Select side AB. From the **Construct** menu, choose **Midpoint**.
- Construct line segment CD.

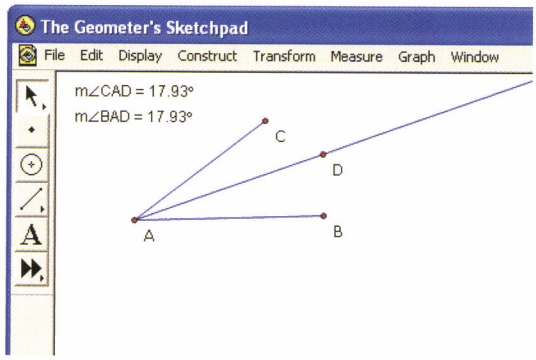
To check that CD is a median:
- Measure the distances AD and DB.

Constructing an Angle Bisector

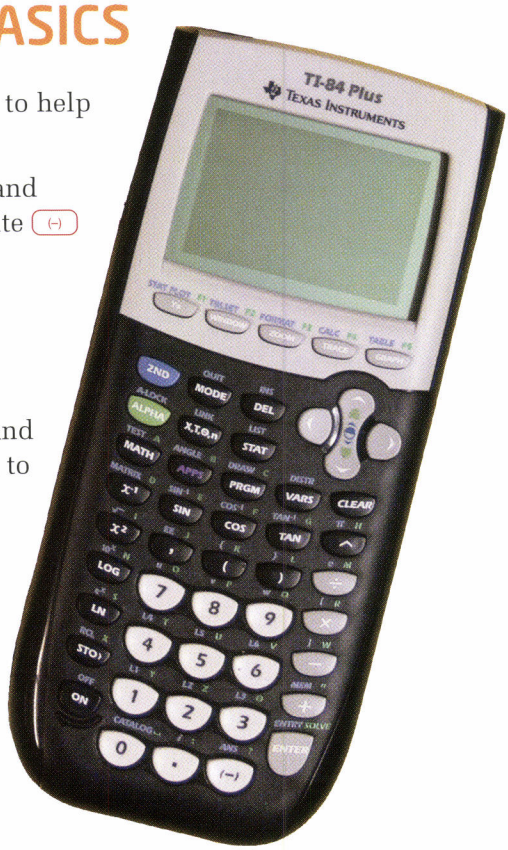
- Use the **Straightedge Tool** to construct line segments AB and AC to form ∠CAB.
- Select the points C, A, and B, in that order.
- From the **Construct** menu, choose **Angle Bisector**. The angle bisector will appear.

To verify that this is the angle bisector:

- Construct a point D on the angle bisector.
- Measure angles CAD and BAD.



TI-83 PLUS AND TI-84 PLUS BASICS



The keys on the TI-83 Plus and TI-84 Plus are colour-coded to help you find the various functions.

- The white keys include the number keys, decimal point, and negative sign. When entering negative values, use the white (−) key, not the grey − key.
- The grey keys on the right side are the math operations.
- The grey keys across the top are used when graphing.
- The primary function of each key is printed on the key, in white.
- The secondary function of each key is printed in yellow and is activated by pressing the yellow 2nd key. For example, to find the square root of a number, press 2nd x^2 for [√_].
- The alpha function of each key is printed in green and is activated by pressing the green ALPHA key.

Adjusting the Contrast

If the calculator screen appears unclear, you can adjust the contrast.

- Press 2nd.

To increase the contrast,

- Press and hold ▲ until the desired level is reached.

To decrease the contrast,

- Press and hold ▼ until the desired level is reached.

If you want to change the contrast more slowly, you can repeatedly press 2nd followed by ▲ or 2nd followed by ▼.

Setting the Format

To define a graph's appearance:

- Press 2nd ZOOM for [FORMAT] to see the choices available.

The **Default Settings**, shown here, have all the features on the left "turned on."

```
RectGC PolarGC
CoordOn CoordOff
GridOff GridOn
AxesOn AxesOff
LabelOff LabelOn
ExprOn ExprOff
```

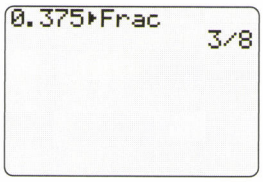
To use Grid Off/Grid On:

- Select [FORMAT] by pressing 2nd ZOOM. Cursor down and right to **GridOn**. Press ENTER.
- Press 2nd MODE for [QUIT].

Displaying a Decimal as a Fraction

- Key in a decimal, such as 0.375.
- Press MATH. Select **1:Frac**.
- Press ENTER.

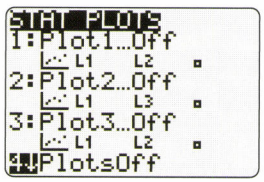
The decimal will be displayed as a fraction.



Turning Stat Plots Off

Before graphing a line, it is wise to ensure that all statistical plots are turned off, or you may see unwanted data on your graph.

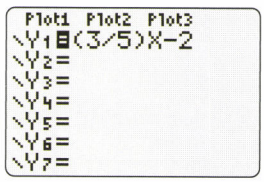
- Press 2nd Y= to access the **STAT PLOTS** menu.
- Select **4:PlotsOff**.

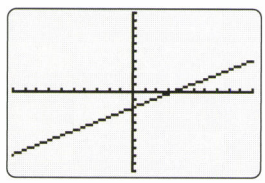


Graphing Linear Relations

- Press Y=. Enter the equation.
- If there are any equations already present in the **Y=** editor, delete them using CLEAR.
- To display the graph, press GRAPH.

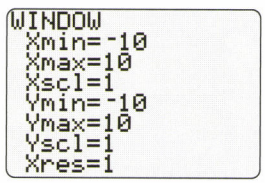
For example, enter $y = \frac{3}{5}x - 2$ by pressing
Y= (3 ÷ 5) X,T,θ,n − 2.
Press GRAPH.





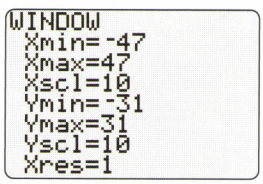
Setting Window Variables

The WINDOW key defines the appearance of the graph.
The standard (default) window settings are shown.



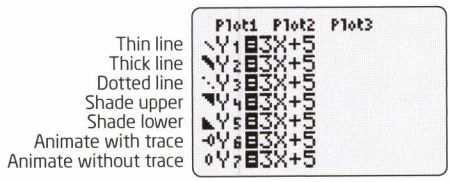
To change the window settings:
- Press WINDOW. Enter the desired window settings.

In the example shown,
- the minimum x-value is -47
- the maximum x-value is 47
- the scale of the x-axis is 10
- the minimum y-value is -31
- the maximum y-value is 31
- the scale of the y-axis is 10
- the resolution is 1, so equations are graphed at each horizontal pixel



Changing the Appearance of a Line

The default graph style is a thin, solid line. The line style is displayed to the left of the equation. There are seven options for the appearance of a line.



- Press Y= and clear any equations there from previous use of the calculator.
- Enter the relation $y = 3x + 5$ for **Y1**.
- Use the standard window settings.
- Press GRAPH.

Note the thin, solid line style.

- Press Y=. Cursor left to the slanted line.
- Press ENTER repeatedly until the thick, solid line shows, as in **Y2** above.
- Press GRAPH.

Note the thick, solid line.

- Press Y=. Cursor left to the slanted line.
- Press ENTER repeatedly until the thin dotted line shows, as in **Y3** above.
- Press GRAPH.

Note the thin, dotted line.

- Press Y=. Cursor left to the slanted line.
- Press ENTER repeatedly until the solid triangle with hypotenuse down shows, as in **Y4** above.
- Press GRAPH.

Note that the graph is shaded above the line.

- Press Y=. Cursor left to the slanted line.
- Press ENTER repeatedly until the solid triangle with hypotenuse up shows, as in **Y5** above.
- Press GRAPH.

Note that the graph is shaded below the line.

- Press Y=. Cursor left to the slanted line.
- Press ENTER repeatedly until the circle with a tail shows, as in **Y6** above.
- Press GRAPH.

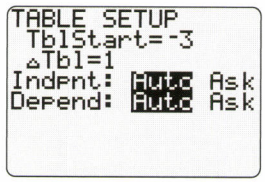
Note the flying ball tracing the graph.

- Press Y=. Cursor left to the slanted line.
- Press ENTER repeatedly until the plain circle shows, as in **Y7** above.
- Press GRAPH.

Note the flying ball that follows the graph, but does not leave a trace.

Setting Up a Table of Values

To set up a table of values, specifying a starting x-value and an x-increment:
- Press [Y=]. Enter a relation, such as $2x + 1$.
- Press [2nd] [WINDOW] for [TBLSET].

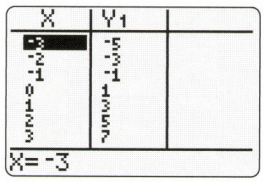


Enter the desired starting x-value for **TblStart**, such as -3.

Enter the desired x-increment for **ΔTbl**, such as 1.

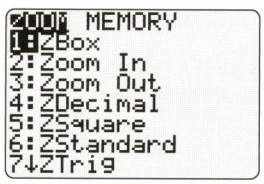
Ensure that **Indpnt** and **Depend** are both set to **Auto**.

- Press [2nd] [GRAPH] for [TABLE] to view the table.



Using Zoom

The [ZOOM] key is used to change the area of the graph that is displayed in the graphing window.



To set the size of the area you want to zoom in on:
- Press [ZOOM]. Select **1:Zbox**. The graph screen will be displayed, and the cursor will be flashing.
- If you cannot see the cursor, use the [▶], [◀], [▲], and [▼] keys to move the cursor until you see it.
- Move the cursor to an area on the edge of where you would like a closer view. Press [ENTER] to mark that point as a starting point.
- Press the [◀], [▶], [▲], and [▼] keys as needed to move the sides of the box to enclose the area you want to look at.
- Press [ENTER] when you are finished. The area will now appear larger.

To zoom in on an area without identifying a boxed-in area:
- Press [ZOOM]. Select **2:Zoom In**.

To zoom out of an area:
- Press [ZOOM]. Select **3:Zoom Out**.

To display the viewing area where the origin appears in the centre and the x- and y-axes intervals are equally spaced:
- Press [ZOOM]. Select **4:ZDecimal**.

To reset the axes ranges on your calculator to the standard window:
- Press [ZOOM]. Select **6:ZStandard**.

To display all data points in a **STAT PLOT:**
- Press [ZOOM]. Select **9:ZoomStat**.

Tracing a Graph

- Enter a function using [Y=].
- Press [TRACE].
- Press [◀] and [▶] to move along the graph.
- You can also input an x-value when you are in TRACE, to get exact y-values.

The x- and y-values are displayed at the bottom of the screen.

If you have more than one graph plotted, use the [▲] and [▼] keys to move the cursor to the graph you wish to trace.

You may want to turn off all STAT PLOTS before you trace a function:
- Press [2nd] [Y=] for [STAT PLOT]. Select **4:PlotsOff**.
- Press [ENTER].

Using the Intersect Operation

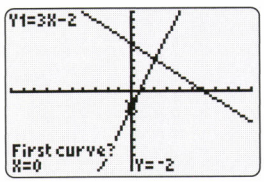
The **intersect** operation will find the point at which two relations intersect.

For example, consider the lines $y = 3x - 2$ and $y = -x + 6$.

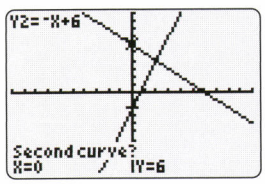
Turn off all STAT PLOTS.
- Press [Y=]. Clear any relations already present.

Enter the relations in **Y1** and **Y2**.
- Press [ZOOM]. Select **6:ZStandard**.
- Press [2nd] [TRACE] to access the **CALCULATE** menu. Select **5:intersect**.

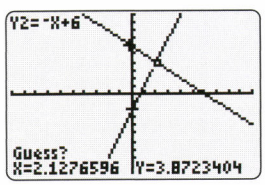
If necessary, use the cursor keys to select the first relation.
- Press [ENTER].



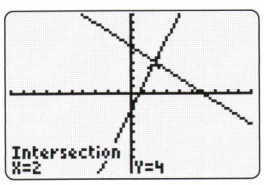
If necessary, use the cursor keys to select the second relation.
- Press [ENTER].



Use the cursor keys to position the cursor as close as possible to your guess for the point of intersection.
- Press [ENTER].



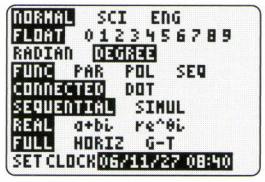
The coordinates of the point of intersection will be displayed.



Changing the Mode Settings

The mode settings are used to control the way the calculator displays and interprets numbers and graphs.

- Press MODE.



The first line controls the number display. Most of the time, you will use **Normal**. You can also select **Scientific** or **Engineering**.

The second line controls the number of decimal places that are displayed. If you choose **FLOAT**, the calculator will select the appropriate number. You can also choose the number of decimal places. For example, if you are working with money, you might want to have all numbers displayed with two decimal places.

The third line selects angle measures as **RADIAN** or **DEGREE**. You will normally use degree measure.

The bottom line lets you set the clock on a TI-84 Plus calculator. Note: The TI-83 Plus does not have a clock.

To change a setting:
- Use the cursor keys to navigate to the desired setting.
- Press ENTER to select the setting.

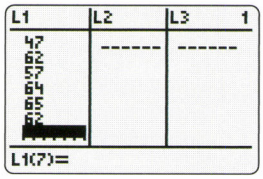
To leave the mode screen:
- Press 2nd MODE for [QUIT].

Entering Data Into Lists

To enter data:
- Press STAT. The cursor will highlight the **EDIT** menu.
- Press 1 or ENTER to select **1:Edit...**.

This allows you to enter new data, or edit existing data, in lists **L1** to **L6**.

For example, press STAT, select **1:Edit...**, and then enter six test scores in list **L1**.
- Use the cursor keys to move around the editor screen.
- Complete each data entry by pressing ENTER.
- Press 2nd MODE for [QUIT] to exit the list editor when the data are entered.



You may need to clear a list before you enter data into it. For example, to clear list **L1**:
- Press STAT and select **4:ClrList**.
- Press 2nd 1 for [L1], and press ENTER.

OR

To clear all lists:
- Press 2nd + for [MEM] to display the **MEMORY** menu.
- Select **4:ClrAllLists**, and then press ENTER.

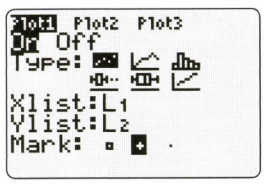
Creating a Scatter Plot

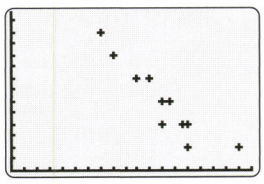
To create a scatter plot:

- Enter the two data sets in lists **L1** and **L2**.
- Press 2nd Y= for [STAT PLOT].
- Press 1 or ENTER to select **1:Plot1...**.
- Press ENTER to select **On**.
- Cursor down, and then press ENTER to select the top left graphing option, a scatter plot.
- Cursor down and press 2nd 1 for [L1].
- Cursor down and press 2nd 2 for [L2].
- Cursor down and select a mark style by pressing ENTER.
- Press 2nd MODE for [QUIT] to exit the **STAT PLOTS** editor when you are finished.

To display the scatter plot:

- Press Y= and use the CLEAR key to remove any graphed equations.
- Press 2nd MODE for [QUIT] to exit the **Y=** editor.
- Press ZOOM and select **9:ZoomStat** to display the scatter plot.

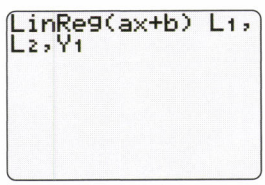


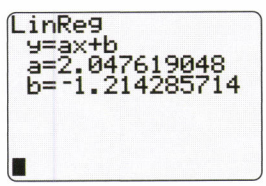


Line of Best Fit: Linear Regression

You can add the line of best fit to a scatter plot by using the **LinReg** function.

- Create a scatter plot.
- Press STAT. Cursor over to the **CALC** menu. Then, select **4:LinReg(ax+b)**.
- Press 2nd 1 for [L1], followed by , .
- Press 2nd 2 for [L2], followed by , .
- Press VARS. Cursor over to the **Y-VARS** menu. Select **1:FUNCTION**, and then press ENTER. Select **1:Y1** and press ENTER.
- Press ENTER to obtain the **LinReg** screen. The equation of the line is displayed.

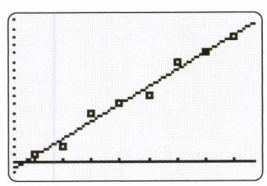




Note: If the diagnostic mode is turned on, you will see values for **r** and $\mathbf{r^2}$ displayed on the **LinReg** screen. To turn the diagnostic mode off:

- Press 2nd 0 for [CATALOG].
- Scroll down to **DiagnosticOff**. Press ENTER to select this option.
- Press ENTER again to turn off the diagnostic mode.

Press GRAPH to see the line of best fit overlaid on the scatter plot.

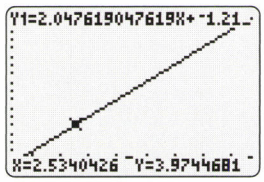


The linear regression equation is stored in the **Y=** editor. Press Y= to display the equation generated by the calculator.

You can use the line of best fit to estimate values.

Turn STAT PLOTs off.

- Press GRAPH.
- Press TRACE.

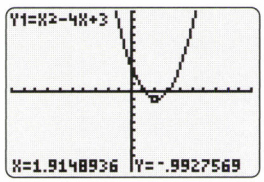


Use the cursor keys to move the cursor back and forth along the line of best fit to estimate values.

Finding the Maximum or Minimum

To find the maximum or minimum of a relation such as $y = x^2 - 4x + 3$:

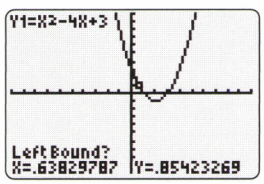
- Press Y=. Enter the relation.
- Press ZOOM. Select **6:ZStandard**.
- Press TRACE. Use the cursor keys to find the minimum value for y.



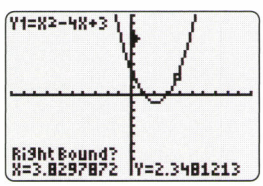
The TRACE method may not give an exact answer.

You can obtain a more accurate value for the maximum or minimum as follows:

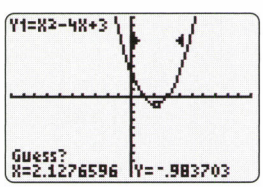
- Press 2nd TRACE to access the **CALCULATE** menu. Select **3:minimum**.
- Use the cursor keys to move the cursor to the left of the minimum. Press ENTER.



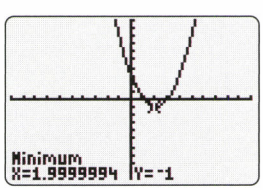
- Use the cursor keys to move the cursor to the right of the minimum. Press ENTER.



- Use the cursor keys to move the cursor close to your guess for the minimum. Press ENTER.



- The coordinates of the minimum are displayed.

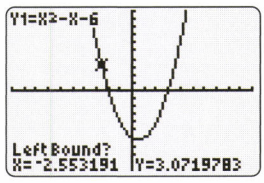


If the graph has a maximum, rather than a minimum, select **4:maximum** from the **CALCULATE** menu.

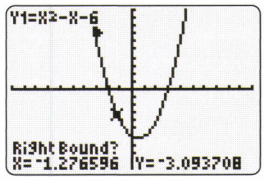
Finding Zeros

To find the zeros of a relation such as $y = x^2 - x - 6$:

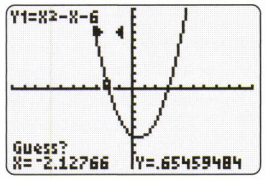
- Press Y=. Enter the relation.
- Press ZOOM. Select **6:ZStandard**.
- Press 2nd TRACE to access the **CALCULATE** menu. Select **2:zero**.



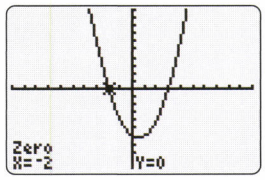
- Use the cursor keys to move the cursor to the left of the left zero.
- Press ENTER.



- Use the cursor keys to move the cursor to the right of the left zero.
- Press ENTER.



- Use the cursor keys to move the cursor close to your guess for the left zero.
- Press ENTER.



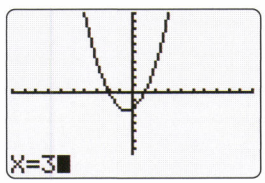
The coordinates of the zero are displayed.

Use a similar procedure to find the right zero.

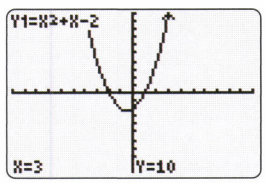
Using the Value Operation

To find the corresponding y-value for any x-value for a relation such as $y = x^2 + x - 2$:

- Press Y=. Enter the relation.
- Press ZOOM. Select **6:ZStandard**.
- Press 2nd TRACE to access the **CALCULATE** menu. Select **1:value**.
- Enter a value for x, such as $x = 3$.
- Press ENTER.



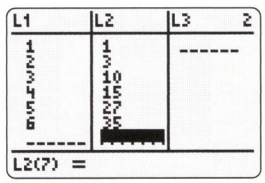
The corresponding y-value, $y = 10$, is displayed.

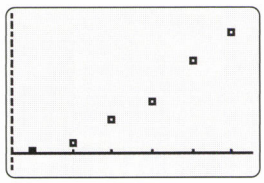


Curve of Best Fit: Quadratic Regression and Exponential Regression

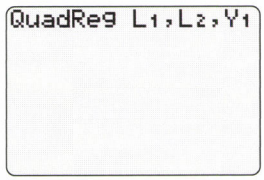
You can add a curve of best fit to a scatter plot by using the QuadReg operation to try a quadratic fit, or the ExpReg operation to try an exponential fit.

Create a scatter plot using the data shown.

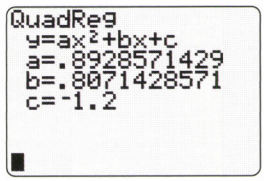




- Press STAT. Cursor over to the **CALC** menu. Then, select **5:QuadReg**.
- Press 2nd 1 for L1, followed by ,.
- Press 2nd 2 for L2, followed by ,.
- Press VARS. Cursor over to the **Y-VARS** menu. Select **1:Function**, and press ENTER. Select **1:Y1**.



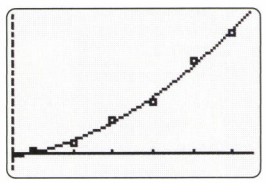
- Press ENTER to obtain the **QuadReg** screen. The equation of the quadratic curve is displayed.



Note: If the diagnostic mode is turned on, you will see values for **r** and $\mathbf{r^2}$ displayed on the **QuadReg** screen. To turn the diagnostic mode off:

- Press 2nd 0 for [CATALOG].
- Scroll down to **DiagnosticOff**. Press ENTER to select this option.
- Press ENTER again to turn off the diagnostic mode.

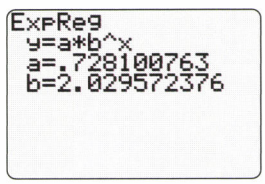
Press GRAPH to see the curve of best fit overlaid on the scatter plot.



The quadratic regression equation is stored in the **Y=** editor.

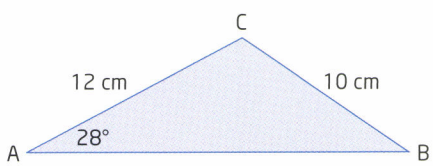
- Press Y= to display the equation generated by the calculator.

If you want to fit an exponential curve, select **0:ExpReg** from the **STAT CALC** menu, rather than **5:QuadReg**.



Calculations Involving the Sine Law

For example, consider △ABC. Find the measure of ∠B and the length of side *c*.

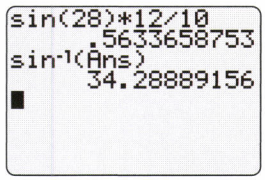


Use the sine law to find the measure of ∠B:

$$\sin B = \frac{\sin 28°}{10} \times 12$$

To solve for ∠B:

- Ensure that the **MODE** is set to **Degree**.
- Enter SIN 28) × 12 ÷ 10 ENTER.
- Press 2nd SIN. This will access the [SIN^{-1}] function to calculate the angle.
- Press 2nd (−) for [ANS]. This will insert the answer from the previous calculation.
- Press) and then ENTER.



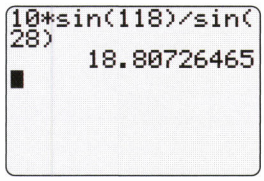
The measure of ∠B is about 34°.

Use the sine law to find the length of side c:

$$c = \frac{10 \sin 118°}{\sin 28°}$$

Solve for c:

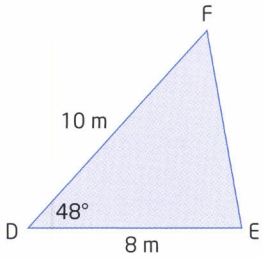
- Press 10 × SIN 118) ÷ SIN 28) ENTER.



The length of side AB is about 18.8 cm.

Calculations Involving the Cosine Law

Consider △DEF. Find the length of EF and the measure of ∠E.

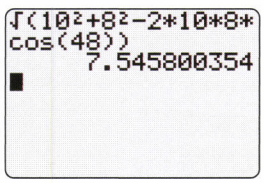


Use the cosine law to find d:

$$d^2 = e^2 + f^2 - 2ef \cos D$$
$$= 10^2 + 8^2 - 2 \times 10 \times 8 \times \cos 48°$$
$$d = \sqrt{10^2 + 8^2 - 2 \times 10 \times 8 \times \cos 48°}$$

To evaluate d:

- Press 2nd x^2 for [√].
- Press (10 x^2 + 8 x^2 − 2 × 10 × 8 × COS 48)).
- Press ENTER.



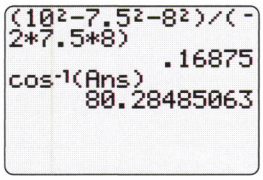
The length of EF is about 7.5 m.

Use the cosine law to find ∠E:

$$e^2 = d^2 + f^2 - 2df \cos E$$
$$10^2 = 7.5^2 + 8^2 - 2 \times 7.5 \times 8 \times \cos E$$
$$10^2 - 7.5^2 - 8^2 = -2 \times 7.5 \times 8 \times \cos E$$
$$\cos E = \frac{10^2 - 7.5^2 - 8^2}{-2 \times 7.5 \times 8}$$

To solve for ∠E:

- Press (10 x^2 − 7.5 x^2 − 8 x^2) ÷ ((−) 2 × 7.5 × 8).
- Press ENTER.
- Press 2nd COS. This will access the COS^{-1} function to calculate the angle.
- Press 2nd (−) for [ANS]. This will insert the answer from the previous calculation.
- Press) and then ENTER.



The measure of ∠E is about 80°.

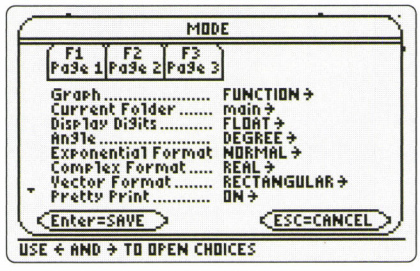
TI-89 TITANIUM COMPUTER ALGEBRA SYSTEM BASICS

A Computer Algebra System, or CAS, is a program that contains tools for manipulation of symbolic expressions. It is sometimes called a symbolic manipulator. You can expand expressions, factor expressions, solve equations, check solutions, and perform many other operations with a CAS.

Keypad Tips: Use the 2ND key to access functions that are in **blue**. Use the ♦ key to access functions that are in **green**. Use the ALPHA key to access letters and other text characters. Use the (-) key to enter a negative, but use the − key for the subtraction operation.

Changing the Mode Settings

The mode settings are used to control the way the calculator displays and interprets numbers and other quantities.



- Press MODE.

The **Display Digits** line determines the number of decimal places that will be displayed. If you choose **FLOAT**, the calculator will select the appropriate number of decimal places. You can also choose the number of decimal places. For example, if you are working with money, you might want to have all numbers displayed with two decimal places.

The **Angle** line selects the type of angle measure used by the calculator. You will normally use degree measure.

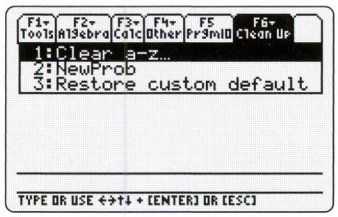
The remaining lines should be set as shown, unless you are instructed otherwise.

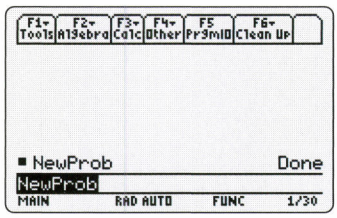
- To leave the mode screen, press ESC.

Starting a New Problem

Before you start using the calculator, it is wise to clear the memory. Otherwise, values that a previous user has entered may produce unexpected results.

- Press [2nd] [F1] to access the **F6** menu. This is known as the **Clean Up** menu.
- Select **2:NewProb**. Then, press [ENTER].


F1 Tools F2 Algebra F3 Calc F4 Other F5 PrgmIO F6 Clean Up
1:Clear a-z...
2:NewProb
3:Restore custom default
TYPE OR USE ↔↑↓ + [ENTER] OR [ESC]



Operations on Equations

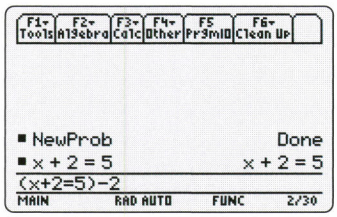
The syntax for CAS operations is somewhat different from that usually used in pencil and paper calculations.

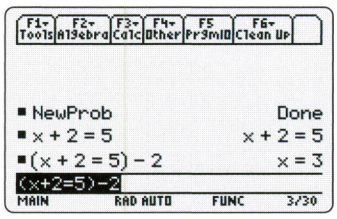
For example, consider solving the equation $x + 2 = 5$. When using pencil and paper, you subtract 2 from both sides.

$x + 2 - 2 = 5 - 2$

When using a CAS, you place the entire equation in brackets, and then indicate the subtraction operation outside the brackets, as shown.

- Press [ENTER] to finish the operation.





Copying and Pasting

You can copy a previous entry in the workspace and paste it into the command line. This is useful for several purposes, such as when solving an equation.

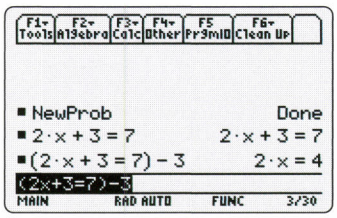
As an example, consider a step-by-step solution of the equation $2x + 3 = 7$, as shown.

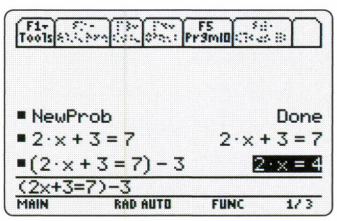
- Enter the equation $2x + 3 = 7$.
- Subtract 3 from both sides.
- Press [ENTER].

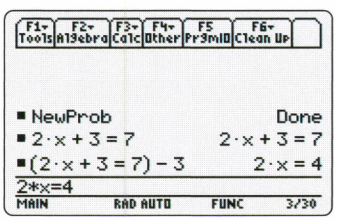
For the next step, you want to divide the intermediate result by 2.

- Use the cursor key to move up to $2x = 4$.
- Press [♦] [↑] for [COPY].
- Move the cursor back to the command line.
- Press [♦] [ESC] for [PASTE].

Note that $2x = 4$ is pasted into the command line.

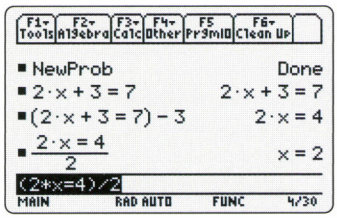




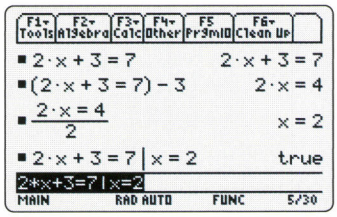


- Insert brackets around $2x = 4$, and divide by 2.
- Press ENTER.

The solution is $x = 2$.


F1 Tools F2 Algebra F3 Calc F4 Other F5 PrgmIO F6 Clean Up
NewProb Done
2·x + 3 = 7 2·x + 3 = 7
(2·x + 3 = 7) − 3 2·x = 4
2·x = 4 / 2 x = 2
(2*x=4)/2
MAIN RAD AUTO FUNC 4/30

To check the solution:
- Use the cursor to move up to the original equation, $2x + 3 = 7$.
- Press ♦ ↑ for [COPY].
- Move the cursor back to the command line.
- Press ♦ ESC for [PASTE].
- Type | $x = 2$.
- Press ENTER.


F1 Tools F2 Algebra F3 Calc F4 Other F5 PrgmIO F6 Clean Up
2·x + 3 = 7 2·x + 3 = 7
(2·x + 3 = 7) − 3 2·x = 4
2·x = 4 / 2 x = 2
2·x + 3 = 7 | x = 2 true
2*x+3=7|x=2
MAIN RAD AUTO FUNC 5/30

Note that the CAS returns a **true** message. This means that $x = 2$ is a solution.

For a simple equation such the one shown in this example, it may be just as fast to retype each time. However, COPY and PASTE can be very useful for more complicated expressions.

Note: The calculator remembers your entries even if they have "disappeared" off the top of the work space. You can cursor upward, and they will reappear. They will remain until you clear the memory or until you fill up all available memory.

Using the CLEAR Key and the Backspace Key

When the cursor is in the command line:
Pressing ←, the backspace key, will delete the character immediately to the left of the cursor. You can use it to correct typos.

Pressing CLEAR will delete the entire contents of the command line. You can use it to clear the command line.

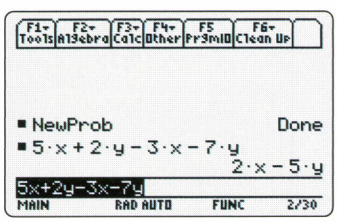
When the cursor is in the work space, pressing either key will delete the line that the cursor is on.

Collecting Like Terms

A CAS can collect like terms in an algebraic expression.

Consider the expression $5x + 2y - 3x - 7y$.

- Type this expression into the calculator.
- Press ENTER.



Note that the CAS has collected and combined like terms.

Tip: The variables x, y, z, and t have dedicated keys. Other variables can be accessed using ALPHA. When entering a term such as $5x$, you do not need to enter a multiplication operator between the 5 and the x. However, if you enter a term such as xy, the situation is different. If you do not enter a multiplication operator between the x and the y, the CAS considers this as a single variable named xy, not as the product of x and y. If you want the term to mean the product of x and y, you must enter a multiplication operator between the two letters.

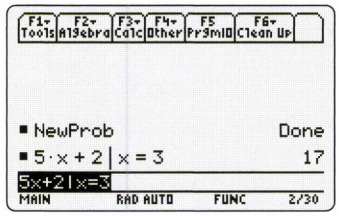
Evaluating an Expression for a Given Value

You can evaluate an expression for a given value of the variable.

For example, consider the expression $5x + 2$.

To evaluate the expression for $x = 3$:
- Type $5x + 2$ (|) $x = 3$.
- Press (ENTER).

The calculator returns a value of 17.



Checking the Solution for an Equation

You can use a CAS to check whether a given value is the correct solution for an equation.

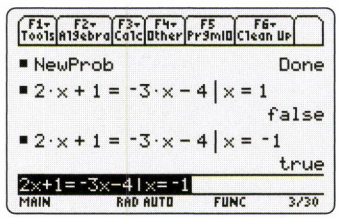
For example, consider the equation $2x + 1 = -3x - 4$. Check $x = 1$ and $x = -1$ as possible solutions.

Check $x = 1$:
- Type $2x + 1 = -3x - 4$ (|) $x = 1$.
- Press (ENTER).

Note that the CAS returns a **false** message. This means that $x = 1$ is not a solution.

Check $x = -1$:
- Type $2x + 1 = -3x - 4$ (|) $x = -1$.
- Press (ENTER).

Note that the CAS returns a **true** message. This means that $x = -1$ is a solution.



Using the Expand Operation

You can use a CAS to expand expressions.

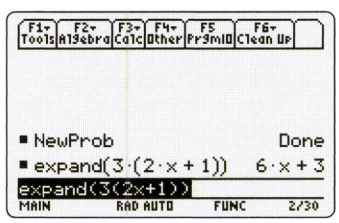
For example, consider the expression $3(2x + 1)$.

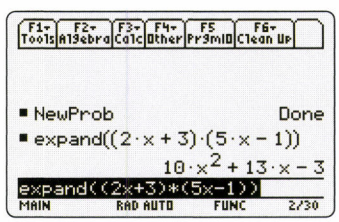
- Press (F2) and select **3:expand(**.
- Type in the expression and a closing bracket.
- Press (ENTER).

Note that the CAS has expanded the expression using the distributive property.

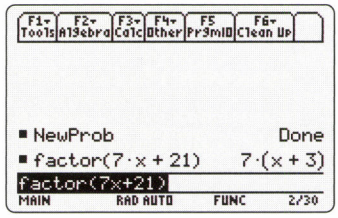
The CAS will expand any algebraic expression.

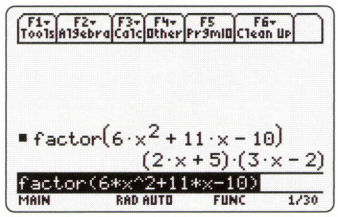
For example, try $(2x + 3)(5x - 1)$.





Using the Factor Operation





You can use a CAS to factor expressions.

For example, consider the expression $7x + 21$.

- Press F2 and select **2:factor(**.
- Type in the expression and a closing bracket.
- Press ENTER.

Note that the CAS has factored the expression by taking out a common factor.

The CAS will factor other algebraic expressions.

For example, try $6x^2 + 11x - 10$.